A Computational Approach to the Phonology of Connected Speech

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School of Oriental & African Studies, University of London

To my parents, who took me to see the Rosetta Stone.

And to Alex.

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Abstract

This thesis attempts to answer the question 'How do we store and retrieve linguistic information?', and to show how this is intimately related to the question of connected speech phonology. The main discussion begins in Chapter One with a non-linguistic introduction to the problem of looking things up, and considers in particular the *hashtable* and its properties. The theme is developed directly in the latter part of the chapter, and further in Chapter Two, where it is proposed not only that the hashtable is the mechanism actually used by the language faculty, but also that phonology is that mechanism. Chapter Two develops in detail a radically new theory of phonology based on this hypothesis, and examines at length its ramifications.

As a foundation for understanding how the phonological and the conceptual-semantic forms of utterances are related, we undertake a detailed study of the relationship between 'form' and 'meaning' in Chapter Three. We propose a general algorithm, which we claim is a real mechanism driving the acquisition of morphological knowledge, that can abstract and generalise these sorts of morphological relationships. We examine its computational properties, which are surprisingly favourable, and provide a detailed quasi-experimental case-study.

By Chapter Four, all the theoretical necessities for describing and explaining what are traditionally believed to be phonological processes operating at the level of the sentence have been introduced. The chapter is used to show how the pieces of Chapters One, Two and Three fit together to tell this story. The chapter also offers some well-motivated speculation on new lines research suggested by some of the computational results obtained throughout this work, and provides a meta-level framework for the future development of a full-scale theory of syntactic function and its acquisition.

Contents

Foreword

A teacher of mine once told me that the secret of knowledge is not knowing something, but knowing where to look it up. As I was then a teenager about to be faced with examinations in Classical Greek and Latin, this was of little immediate comfort. The work that has culminated in this dissertation has been to discover and explain how information is encoded and communicated in speech, by human beings, in 'real' time. It must, therefore, take special care to examine how this information is *stored* and how it can be *accessed*. That is to say, this work is not just about what we know, but also about how we know where to look it up.

The results of studying the problem from this perspective are surprising, and seriously challenge many, if not all, the assumptions commonly made by linguists about the 'division of labour' within the language system, and the properties these divisions ought or ought not to possess. It is widely held, even by most phonologists, for example, that lips and tongues are relevant to language, or that the phonological component of our linguistic knowledge is an interface to 'the outside world'; it is a commonplace assumption that the lexicon contains only idiosyncratic information; the question at the heart of this study is *never* addressed.

Inspiration during the course of this project has come from many sources, from my classical and mathematical schooling, from my passion, since childhood, for language 'collecting', from the diversity of specialisations in the Department of Linguistics at SOAS. The merciless approach we phonologists take to each other's work during workshops and seminars at SOAS has provided a particularly compelling atmosphere in which to work.

The ultimate inspirational debt is to Jonathan Kaye who had the first inkling that this was going to be an exciting area of study, and convinced me likewise. I don't think either of us expected the far reaching consequences, but that has been half the fun.

Preface

> In pursuing a minimalist program, we want to make sure that we are not inadvertently sneaking in improper concepts, entities, relations, and conventions.
>
> — *Chomsky* 1995:225

This dissertation is very much an exploration. The work is concerned more with a programme for investigation than with the minutiæ of a particular theoretical problem and its analysis. In this sense I am aiming at a level of explanation beyond that usually encountered in the day-to-day work of phonologists and morphologists. The navel-gazing which is essential to such revisionism inevitably requires setting aside most of what has come to be held dear, and starting with a blank sheet of paper. To those who are sceptical of re-inventions of wheels as a viable methodology I would ask 'how do you know your wheel is round?' In this work I hope to have shown that it is possible, and desirable, to develop a theory of roundness that allows us to make wheels which we *know* to be round.

The main discussion begins in Chapter One with a non-linguistic introduction to the problem of looking things up, and considers in particular the *hashtable* and its properties. The theme is developed directly in the latter part of the chapter, and further in Chapter Two, where it is proposed not only that the hashtable is the mechanism actually used by the language faculty, but also that the phonology is that mechanism. Chapter Two develops in detail a radically new theory of phonology based on this hypothesis, and examines at length its ramifications.

As a foundation for understanding how the phonological and the conceptual-semantic forms of utterances are related, we undertake a detailed study of the relationship between 'form' and 'meaning' in Chapter Three. We propose a general algorithm, which we claim is a real mechanism driving the acquisition of morphological knowledge, that can abstract and generalise these sorts of morphological relationships. We examine its computational properties, which are surprisingly favourable, and provide a detailed quasi-experimental case-study.

By Chapter Four, all the theoretical baggage necessary for describing, and, I hope, explaining, what are traditionally believed to be phonological processes operating at the level of the sentence, has been introduced. The chapter is used to show how the pieces of Chapters One, Two and Three fit together to tell this story. The chapter also offers some well-motivated speculation on new lines research suggested by some of the computational results obtained throughout this work, and provides a meta-level framework for the future development of a full-scale theory of syntactic function and its acquisition.

The work, I hope, shows how close we may actually be to realising a computationally attractive algorithmic characterisation of connected speech processing and language acquisition.

Introduction

> I only wish to suggest that B&H's assumptions presented in (49) above are not *a priori* true. Since they are unaccompanied by any form of argumentation I feel justified in dismissing them.
>
> —*Kaye* 1995*a*:319–20

A major obstacle to the development of attractive computational implementations of models of human language seems to be the intrinsic computational complexity of the theoretical models. In these days of exponential growth of computing power, my lay friends are often puzzled at the apparent lack of progress in truly human-like language-enabled hardware and software. It is the task of this thesis to show that it is possible to build a realistic theoretical model of human language which is not prohibitively complex. This involves, basically, discarding most of the formal appartus employed by linguists today, and rebuilding from first principles. The chief focus of this thesis, is, therefore, the hypothesis that there are no phonological rules for connected speech.

Despite the appearance of the word 'computational' in the title of this work, the investigation is extremely far-reaching. In demonstrating the feasability of our programme, we have found it necessary to question some of the most deep-rooted assumptions and methodologies that pervade the field today. In so doing we have uncovered a serious logical flaw in accepted phonological metatheory, and have found many other undermotivated *a priori*-isms.

However, although thoroughly radical, our programme is not 'off the wall'. We have endeavoured as much as possible to introduce assumptions which are already well accepted, or at least whose plausibility is easily demonstrated. In this sense we try to use assumptions which are minimally *necessary* for any and all theories of human language. Part of the process of developing a non-complex theory of human language has been to show that these minimal conditions may also in fact be *sufficient*.

The first step in demonstrating that there is no connected speech phonology is to show that phonology itself is not rule (or constraint, or derivation) based. For if there are no rules in the phonology, it is that much harder to justify their introduction simply to 'account for' apparent connected speech phenomena.

Chapter One shows that phonology has an unexpected role in the design of the human linguistic system. This role comes to light during the investigation of whether or not phonological rules might be motivated by scarce computational resources, as is commonly claimed.

In Chapter Two we prove that there are some unexpected properties of rule-based theories which pose insurmountable methodological difficulites. The generality of the proof makes it valid for rules of connected speech phonology, too.

However, this new world view comes with no small price tag. The remainder of the Chapter, and the ensuing Chapters serve to demonstrate that those residual phenomena which are claimed to support rule-based theories can in fact be explained, more insightfully in our opinion, in our simplified rule-free theory.

Chapter One

The Lexicon

> First we build the scaffolding. How far beyond the scaffolding we get is an open question; the scaffolding itself can produce an artistic effect deeper than that of the surface alone.
>
> — *Paul Klee, July* 1922[1]

There is no connected speech phonology. This chapter begins the long process of explaining why.

We do not presuppose any particular theoretical framework, but consider a handful of assumptions and empirical observations which are so uncontroversial that they usually 'go without saying'. Those assumptions are:

1. The lexicon stores (linguistic) data.
2. The lexicon can be accessed efficiently (data can be added to and retrieved from the lexicon in 'real time').
3. Human beings communicate using language.
4. *L'arbitraire du signe* (linguistic arbitrariness).

We prove the remarkable result that these assumptions entail that there can only be *exactly one* interface with the lexicon. The chapter continues to explore what this theorem means for other devices, both linguistic and non-linguistic, which need to access the lexicon: namely that at the point where lexical access takes place, all these devices need to provide objects which belong to this single interface.

Given the current state of our knowledge of how the human brain stores information, and our ability to inspect such information, this result might be thought to be largely of theoretical interest, with little hope of verification or falsification. However, we show that from some simple and easily observed facts of human communication (which are, again, so uncontroversial as to be almost trivial), it is possible to demonstrate just what this common interface looks like. In fact, we prove that this single interface *must* consist of phonological representations.

We continue to show how this rather unexpected result is actually rich with predictions. For example it is shown that this architecture entails the possibility of *rebus:* using visual objects to convey linguistic structures, such as using a picture of an eye to convey '1st person subject pronoun' in English.

Throughout, we are painstaking in our logical rigor. This is to reinforce the point that these results, however odd we might be inclined to view them, are inescapable consequences of the assumptions listed above.

The structure of the argumentation is as follows. Sections 1.1 to 1.3 establish some basic properties of the linguistic lexicon, based on general considerations of storage (§1.1) and access (§1.2; addressing assumption 1); efficient access within physical resource constraints (§1.3; addressing assumption 2); and the location of human language within the spectrum of these access mechanisms (§1.4; addressing assumption 3). The properties of the lexicon thus established, §1.5 demonstrates that linguistic arbitrariness is incompatible with a multi-interfaced lexicon (addressing assumption 4). In §1.6 we explore the consequences for cognitive modules which need to interface with the linguistic lexicon, and show that some commonly observed phenomena such as rebus writing and synonymy follow naturally.

1.1 *Storage*

We highlight in this section that standard assumptions made by linguists about the lexicon fail to stand up under scrutiny, and do not provide an adequate basis for understanding lexical mechanisms.

The considerable questions concerning storage and retrieval of linguistic information, as distinct from the information itself, are seldom, if ever, posed as questions of theoretical interest. The details are assumed to be of interest only to those who actually have to implement a linguistic theory computationally. Indeed, the attitude of much theoretical work is that storage and retrieval is orthogonal to theory[2]—linguistic information may just as well be painted on millions of pingpong balls and stored in a big plastic bag as encoded by the neurons of a human brain or the transistors of a digital computer; a magic bingo caller called 'vocabulary selection' is supposed to exist whose hand can dip into the vast bag of pingpong balls and pull out *le mot juste*, blindfolded, in a matter of milliseconds. In the Introduction to his book *The Minimalist Program*, Chomsky deliberately 'put[s] these matters aside', claiming that 'selection from the lexical repertoire made available in UG' appears to be 'of limited relevance to the computational properties of C_{HL}' (Chomsky 1995:8).

Yet as theoreticians we rely crucially on a rather paradoxical assumption about the storage space for linguistic information that fundamentally shapes the theoretical architectures we propose. Firstly, it must be agreed that lexical storage space is in principle infinite: since any linguistic structure can in principle be idiomatised (or become a *listeme* in the terminology of DiSciullo & Williams 1982), it follows that there is no subset of linguistic structures whose idiomatisation cannot be ruled out. That is, there are no *linguistic* constraints on the number, or (well-formed) structure, of idioms. Any linguistic theory, ***L***, can therefore derive *no* expectations about what structures may or may not be idiomatised and so must assume that it has enough resources available to it to list any subset of the set of linguistic structures. Since the set of linguistic structures $\mathfrak{L}$ is a subset of itself (any set is a subset of itself) ***L*** must therefore behave as if it had the resources available to it to list $\mathfrak{L}$. Given that $\mathfrak{L}$ is defined by a generative grammar, the cardinality of $\mathfrak{L}$ is $\aleph_0$ (countable infinity), so ***L*** must assume that the resources it has available are (countably) infinite. This position is a necessary consequence of the assumption that linguistic structure may have idiosyncratic properties, a property of human language long recognised.

We therefore take the infinitude of the lexicon as the null hypothesis, since any departure from it would require either denying that any linguistic structure can be idiomatised (a move which no linguist to our knowledge has made), or denying that there are an infinite number of linguistic structures (which is demonstrably false).

And herein lies the paradox: theorists typically *do* invoke purported properties of the lexicon, usually as supporting 'evidence' for some architectural facet of their theory, claiming, typically, that 'memory storage and search time are at a premium in the case of language' (Bromberger & Halle 1989:56). This claim should demand considerable justification, given firstly that in principle the size of the lexicon is infinite, and secondly that the state of our knowledge about the physical size of the lexicon and the physical mechanisms used to store even linguistic representations is not currently capable of delivering an empirical statement along the lines of 'it requires *n* neurons to store a lexical entry, and the human brain reserves *N* neurons for storing linguistic information'. Further, given what little we do know about the literally mind-boggling resources available in the human brain,[3] the claim that such resources are 'at a premium' for *any* cognitive system seems at best premature.

Furthermore, once these concepts are accepted they have a direct impact on the proposals made about particular aspects of the structure and behaviour of the grammar. For example, if we try to put as little into the lexicon as possible, we will try to find as many generalisations as possible to extract as much common information as possible. These generalisations, which we may call, loosely, 'rules', will consequently tend to be maximised. Result: a large derivational grammar (with a large computational overhead), and a small lexicon. This position is virtually universal amongst theoreticians.[4] So Chomsky,

> 'I will have little to say about the lexicon here, but what follows does rest on certain assumptions that should be made clear. I understand the lexicon in a rather traditional sense: as a list of 'exceptions', whatever does not follow from general principles. … Assume further that the lexicon provides an 'optimal coding' of such idiosyncrasies.
>
> Take, say, the word *book* in English. … The lexical entry for *book* specifies the idiosyncrasies, abstracting from the principles of UG and the special properties of English. It is the optimal coding of information that just suffices to yield the LF representation and that allows the phonological component to construct the PF representation … .'
>
> — *Chomsky* 1995:235

This belief in the 'paucity of resources' immediately suggests an empirical experiment where the lexicon is taken to and beyond its supposed limits.[5] It would have to be demonstrated that a living subject could reach a point where they could no longer list a linguistic structure. I do not believe there has been any documented (or even anecdotal) evidence that (living) human beings do ever reach, or could be forced to reach, a limit beyond which they can acquire no more (idiosyncratic) linguistic structure.[6]

We certainly do not deny that there are physical constraints on the implementation of the lexicon, and that these constraints may indeed induce some of the structural properties of the lexicon and its access mechanisms. We do not, however, accept *a priori* that these resources are 'at a premium'. Indeed, we take the position that the nature of these constraints is open to investigation (which we undertake in §§1.3–1.4).

The physical brain is, of course, a finite structure, but that does not mean that the grammar has to be aware of that, or even that the behaviour of the grammar should notice that the *physical* lexicon is finite. Human beings are chronologically finite (we die), yet no one has, or would wish to build into the theory of grammar mechanisms which were aware of that, and which constrained the design of the grammar accordingly. The grammar is a device capable of generating an infinite number of infinitely long sentences—*i.e.* its design assumes that whatever resources are needed to process even these infinite sentences are available. Given additionally the logical difficulties (discussed above) in assuming the lexicon not to be infinite, and given the lack of empirical support for such a claim, we feel confident in asserting the null hypothesis that:

LEMMA (1.1). THE INFINITE LEXICON
The size of the lexicon is $\aleph_0$ (countable infinity).

A very real, tangible fact about the human linguistic system is that it works in 'real time', and part of this working involves retrieving information from the (infinitely large) lexicon. Without the prejudice of tradition, this turns out to be a non-trivial question. Although not attracting the attention of theoretical linguists, it has been a subject of fruitful research since the late 1960s and early 1970s amongst computer scientists (Knuth 1975, Sedgewick 1988).[7] This is due largely to the ever increasing power and storage capabilities of digital computers, which make the possibility of storing very large databases on disk a reality. In the next section we address in detail some fundamental concepts of accessing databases relevant to the study of human language.

1.2 *Access*

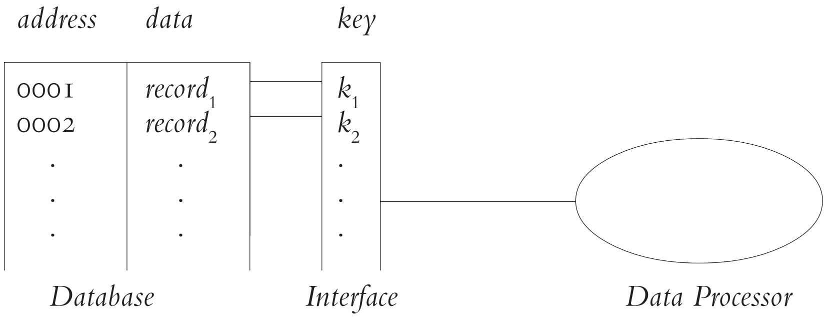
Minimally, a *database* is a set of *records* (or *data*). Each record resides in its own unique area of memory, its *address*. Records are accessed by *keys*. Given any key, the storage and retrieval system can recover the record assigned to that key. In the simplest storage and retrieval system, each record has its own unique key, corresponding to its address. If we imagine the individual residents of a city to be 'records', then each resident ('record') has a postal address ('key') where we can find them (2). In this simple case, each resident has their own unique postal address.

(2) ***Keys and Records***

Key	*Record*	*Postal Address*	*Citizen*
0000	DATA	1 HIGH STREET	FRED
0001	DATA	2 HIGH STREET	JO
0002	DATA	3 HIGH STREET	SAM
.	.	.	.
.	.	.	.
.	.	.	.

This architecture implies that there must be an *interface* mechanism between the database and whatever computational devices manipulate the data stored in the database, so that these computational devices can *access* the data. That is, there must be some (interface) mechanism which can (i) 'go to' an address, and (ii) retrieve (or insert) a record. Diagrammatically this minimal architecture looks like (3).[8]

(3) *The Minimal Storage and Retrieval System*



Now, under the simplest assumptions, the interface has available to it unlimited computational resources. In such a system the interface is able to access any part of the database 'in one go', and for a database of size n, the spatial computational resources required are of order n (written $O(n)$, 'big oh of n'. See *e.g.* Graham, Knuth & Patashnik 1989:443–469).[9] For a finite database Δ of size n, these assumptions are sufficient to build an effective access mechanism: simply allocate resources $C+O(n)$ (see footnote 9, and §1.4). But what if we cannot guarantee that Δ is finite?

If Δ is not finite, then, clearly, infinite computational resources are required, which means that to all intents and purposes there is no *effective* way to access Δ. If the records are scattered arbitrarily throughout Δ (*i.e.* if we have to assume that if the record we are seeking is not under key i, then it could well be under key i+1) then if the record we are seeking is *not* in Δ, the search would continue indefinitely.

Now, assume that there *is* an interface mechanism which behaves exactly as if it *were* an effective procedure, in that it does *not* continue indefinitely if the target record does not exist in Δ. What would that entail? It would simply entail that the records are *not* scattered arbitrarily throughout Δ (*i.e.* there is an arrangement of data in Δ such that if a record r is not under some designated key p then r is not in Δ). An obvious implementation of this would be one in which the records were 'packed' into the database, leaving no empty keys between them (*i.e.* if there is a record under key k then there is a record under key $k-1$, for $k>1$). In this case we can guarantee that if there is no record under key k, then there is no record under key $k+1$, and hence, by induction, that there are no records under any key $k_1 \geq k$. As long as such an empty key p exists, the problem is reduced to one of accessing a *finite* database whose largest key is $p-1$ (requiring resources $C+O(p-1)$, as we have seen). So we can still effectively access an infinite database, as long we assume additionally that (i) the database is packed, (ii) the database is not full (*i.e.* that there exists an empty key in the database).[10]

Given that we assume that human language contains a storage and retrieval system, given the results of §1 that the human language lexicon is infinite, and given that human beings access the lexicon effectively (we do not continue searching indefinitely when trying to look up something which is not in our lexicon), we should expect, minimally, (i) that human language should have an access system which allocates keys such that the lexicon is 'packed', and (ii) that human language should in some way be guaranteed that the lexicon is never full. The investigation of (i) is the subject of the following two sections, the first of which examines the implications of physical constraints on computational resources typical of biological mechanisms, the second of which locates human language within these mechanisms. Corollary (ii) is plausibly guaranteed by the very nature of language acquisition and our physical finiteness—when we are born we do not, under the simplest assumptions, have *any* 'lexical entries', let alone an infinite number of them; and because we are chronologically finite, we could never use our grammars to generate an infinite number of new lexical entries. Thus our linguistic *behaviour* is necessarily finite, so we could only ever increase the content of the lexicon by a finite number of new lexical entries. Assuming the lexicon is packed by the access system, there will consequently always be an empty key.

1.3 *Constraints on Computational Resources*

The architecture in (3), where each key corresponds to each physical address in the database, relies explicitly on the unlimited availability of computing resources. Let us make the not unreasonable assumption that any computational device realised by biological mechanisms is subject to certain resource constraints, determined ultimately by the physics of the mechanism. The crucial aspects of these constraints are that whatever resources are available to the mechanism, they are only available within certain limits (which we might think of as the *tolerance* of the biological mechanism). In other words the behaviour of the mechanism is only guaranteed if these limits are not exceeded. Nature's mechanisms are typically of this kind—human eyes are responsive to light within certain limits; lungs are effective oxygenators just so long as there are certain proportions of certain gases available for inhalation, *etc, etc.*

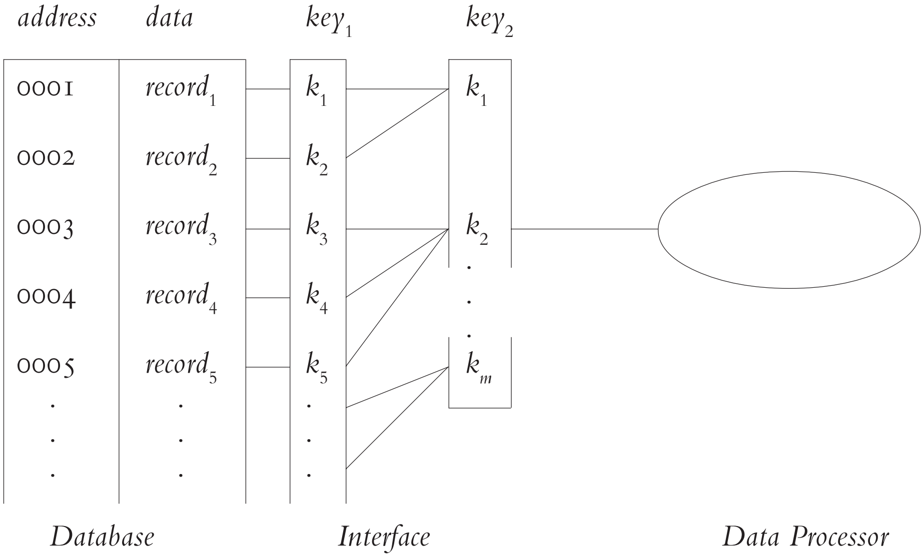
We assume then that a biological mechanism that is required to perform computations has available to it a total (finite) amount of computational resources $R_{max} = \mathrm{S}(A) + T(B)$, where A and B are constants, possibly different for different kinds of mechanism, both dependent ultimately on the *physical* properties of the mechanism. We assume that any computational task requiring spatial resources in excess of $S(A)$, or temporal resources in excess of $T(B)$, exceeds the tolerance of the mechanism. The mechanism, consequently, will be unable to perform the task.

Now, if the architecture in (3) were to be implemented biologically, using resources R, we would have $R \leq \mathrm{S}(A) + T(B)$, and $R = \mathrm{S}(np) + T(1/p)$ (where n is the size of the database, and p the ratio of processors to keys) given by the analysis of the abstract device (3) (see footnote 9). By equating terms we establish the pair of inequalities $\{A \geq np, B \geq (1/p)\}$, which can be simplified straightforwardly by recalling that $p \leq 1$(footnote 9), giving $\{A \geq n, B \geq 1\}$. That is, any biological mechanism instantiating the device (3) can access a database of size no greater than the spatial tolerance A, in time no shorter than one unit, and no greater than the temporal tolerance B.

Now assume that a biological mechanism (let us call it $\mathfrak{M}$) with the same tolerances A and B is called on to access a database which is larger than A. Also assume that it is biologically 'too expensive' to change the structures which actually determine the tolerance of $\mathfrak{M}$. That is, R is still no greater than $\mathrm{S}(A) + T(B)$. How can we alter the internal structure of $\mathfrak{M}$ such that with the same resources R it can access a database of size m, where $m > A$?

The simplest solution is to change the way in which $\mathfrak{M}$ allocates keys to records so that a single key can access more than one record (4).

(4) ***A 'Two-Step' Storage and Retrieval System***



In this way the set of keys can be restricted to some number less than A, which $\mathfrak{M}$ has (spatial) resources to access; having accessed a key, $\mathfrak{M}$ finds a set (or list) of records. This list is in effect a mini-database, so as long as the spatial resources are sufficient to access any one of these mini-databases, $\mathfrak{M}$ will be able to access the database in two steps, using spatial resources no greater than A. The total number of records that this new device can access is therefore the product of the number of new keys and the maximum length of the lists these keys address. Thus for a device which has k keys accessing lists of r records each, the total number of accessible records is $k \times r$. If the spatial resources were divided equally between both stages of the lookup, we would have $k = \frac{1}{2}A$ and $r = \frac{1}{2}A$, giving a maximum potential database size of $\frac{1}{2}A \times \frac{1}{2}A = \frac{1}{4}A^2$ records. However, there is a temporal price to pay, since each lookup is now achieved effectively through *two* lookup operations. So $\mathfrak{M}$ *must* have available temporal resources no greater than B and *no less than* 2 units.

The problem of effectively accessing any one of these mini-databases is exactly the same as that discussed for the simple access device (3). Therefore, if the database is in principle infinite, then any of these mini-databases could in principle be infinite, and so to stand a chance of being accessed effectively they must be packed, and they must contain an empty key_1 (4). That is, the assignment of key_1-s must still pack the database. Note that we do not have to stipulate that the assignment of key_2-s be packed, or contain an empty key, because there are, by definition, a finite number of key_2-s.

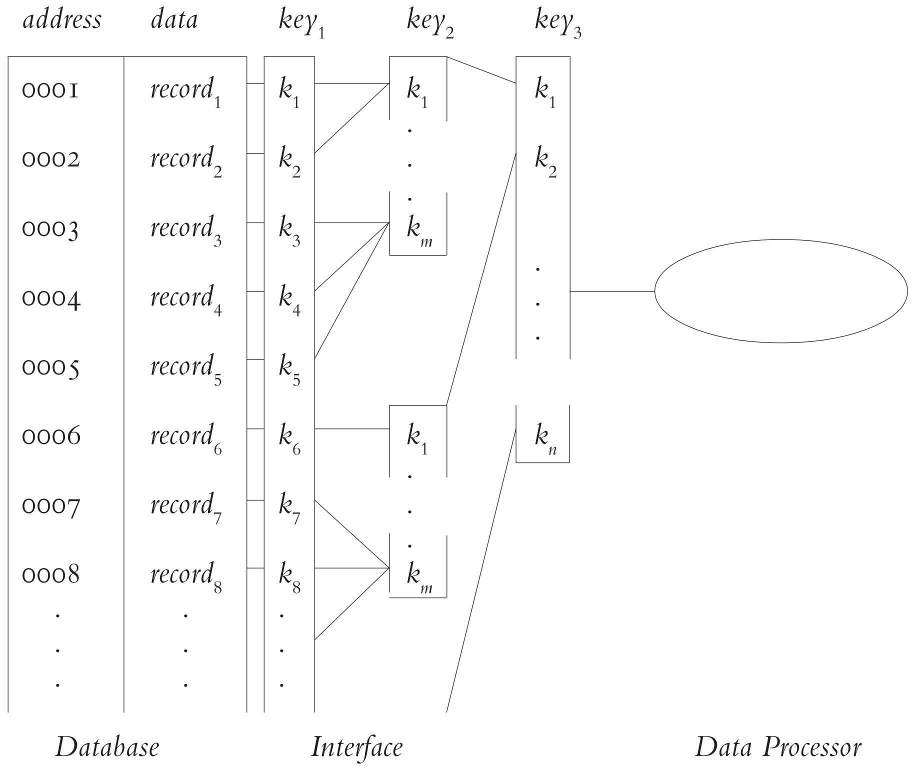
This access strategy has been well studied by computer scientists who know it as *hashing*, since several records are effectively 'hashed together' under a single key (Knuth 1975, Sedgewick 1988). In terms of our town planning analogy, we have the phenomenon of several people being assigned the same postal address (5): a letter arriving at a shared address will need a bit more time before it reaches its recipient, because it must then get from the doormat into the right housemate's hands.

(5) ***Hashing***

Postal Address	*Citizens*
1 HIGH STREET	FRED, NICK, PHIL
2 HIGH STREET	JO, LES
3 HIGH STREET	SAM
.	.
.	.
.	.

Now consider a state of affairs where the database gets so big (*i.e.* contains greater than or equal to ¼A^4 records) that not even mechanism $\mathfrak{M}$ can effectively address it. We can certainly pursue the same design strategy that we did when moving from the simple access device to a hashing device, namely introduce another layer of keys. Consider again the diagram (4), and think of the key_2-s as a mini-database, itself accessed by a key_3. Thus every key_3 accesses a key_2 mini-database, and each key_2 in a given mini-database accesses a packed, non-full mini-database of key_1-s (6).

(6) ***A 'Three-Step' Storage and Retrieval System***



This strategy is commonly referred to as *double-hashing*, where the key_3-s are termed *primary hash keys*, and the key_2-s are termed *secondary hash keys* (Sedgewick 1988). In terms of our town planning analogy this corresponds to the idea of flats (apartments) at each postal address. Thus you can find, say, both SEAN and ALEX at 26 GROVE ROAD (primary hash key), FLAT 3 (secondary hash key). In this case, the maximum size of the database is increased to a potential $(A/3)^3$, with a minimum temporal requirement no less than 3 units of time (for details, see Appendix A).

This design cycle can of course be repeated in principle as often as desired. Let us agree to call the resulting class of mechanisms, $\mathfrak{H}$, *hashing mechanisms*. Let us further agree to call an accessing system that uses *n* layers of keys a *hashing mechanism of order n*, and symbolise the class of all order-*n* hashing mechanisms $\mathfrak{H}(n)$. We can perform the computational resource analysis that we did on $\mathfrak{M}$ recursively to any *n*-order mechanism.[11]

Having now established a minimally simple class of access mechanisms whose computational resources are constrained in a simple, biologically plausible way, we are in a position to try to answer the question of where the human language access system fits into this hierarchy. That is, what order is the human language hashing mechanism? This is the subject of the next section.

1.4 *The Human Language Access Mechanism*

To determine the order of the human language hashing system we need first of all to identify the analogues of the components of the general access device (3) in the human linguistic system. The records of the database (the 'lexicon') we can uncontroversially assume contain, minimally, a morpho-syntactic and semantic specification, whose details we ignore. This is the data that the computational system processes (the 'syntax' and perhaps other post-syntactic devices. Again we ignore the details). Let us introduce some neutral terms for these analogues to facilitate the linguistic discussion. We agree to refer to the *addresses* of the records as *LNodes* (short for 'Lexical Nodes') and the *records* we agree to call *LObjects* (short for 'Lexical Objects'). What, then are the keys?

Consider the act of communicating. Speaker **S** wants hearer **H** to recreate some linguistic structure Σ, the structure which **S** wishes to communicate. Being speakers of the same language, **S** can assume that **H**'s linguistic system, including **H**'s lexicon, is more or less the same as **S**'s. **S** therefore needs to induce **H** to access **H**'s lexicon and recover the linguistic structures needed to build Σ. And we have seen from the discussion in §2 that accessing a database requires an interface device which assigns and manipulates keys to the data. Therefore **H**'s accessing of the lexicon must be performed through keys. In that case, **H** must be able to recover *keys* from whatever **S** communicated; and by definition this key recovery has to be *prior* to lexical access (you can't access a database without a key). Therefore the keys to access the linguistic data in **H**'s lexicon *must* be encoded in the raw material communicated by **S**. But we know what that raw material is—it is the phonological form of **S**'s utterance. In particular, **H** recovers (the phonological forms of) 'words' from this raw material.

This rather simple deduction, arrived at by considering the required 'bare necessities' to implement *any* database access system (§1.2), and by considering the basic facts about human linguistic communication, tells us, then, that the phonological forms of 'words' are keys to access the lexicon.

Although innocent-sounding, this conclusion says something rather striking about the organisation of the human linguistic system—it says that phonological forms are *not* stored in the lexicon. They are, rather, the instruments of accessing the lexicon. This begs the question, could there be other access mechanisms to the lexicon that use, say, 'conceptual-semantic' keys in order to recover phonological forms (thereby implying that phonological forms must also be in the lexicon)? Such a system might seem reasonable from the point of view of utterance production: given that speaker **S** entertains some linguistic structure Σ, how does **S** know which keys (phonological forms) to communicate Σ to **H** with?

There are a number of issues touched on by these questions which are worth pursuing, but we postpone their detailed discussion to §5 below. Whether or not there are other access systems to the lexicon, and whether or not phonological forms could be stored in the lexicon, as well as being keys, does not change the inescapable conclusion that phonological forms *are* keys to the lexicon. Armed with this knowledge we can tackle the question of what order this access system is.

Recall again the simplest lookup system, an $\mathfrak{H}(1)$ system (the 'one-step' access device (3)). The defining property of the interface is that each key accesses exactly one record. If the human language access system were an $\mathfrak{H}(1)$ system then we should expect each key to access exactly one record, that is, each phonological form should access exactly one LObject. But it is trivial to show that this is false. Take the phonological form of almost any word in any language. More often than not this same phonological form gives access to several, often completely unrelated, LObjects. Take for instance a phonological form realised as **ɑj** in English. For English speakers this gives access to LObjects for the '1st person singular subject pronoun' (*I*); 'the ninth letter of the alphabet' (*i*); 'an organ of sight' (*eye*); 'a transitive verb meaning to look at' (*eye*); 'yes' (*aye*). So we must conclude that the human language access system is *at least* $\mathfrak{H}(2)$ (*i.e.* at least a 'two step' system (4)). So the phonological forms of 'words' are *primary hash keys* of the human language access system.

We have now a lower bound for the order of the human language access system, but we have still to establish the order exactly. Consider first what it would mean for human language to be $\mathfrak{H}(2)$. Since there is only one layer of hash keys, and since hash keys are the phonological form of words, we would expect the phonological forms of words to be all of one and the same type. From the properties of hash keys discussed in the sections above it follows that this type is an object which is capable of independent citation (a 'word') and which accesses a *single* list of LObjects (just like English ɑj). But it is clearly false that *all* the phonological forms of words are of just this type. Take another English example, həʊld. On the one hand this form does display the behaviour of a primary hash key: it is capable of independent citation, and it accesses the following list of LObjects—'to keep' (*hold*); 'to contain' (*hold*); 'to be valid' (*hold*); 'a wrestling grip' (*hold*); 'storage area below decks in a ship' (*hold*). On the other hand, there appears to be an additional pattern of access available with həʊld, which is strikingly different. We can actually access *two* lists of LObjects simultaneously with it: one access gives the list 'cavity' (*hole*); 'entirety' (*whole*); 'entire' (*whole*); 'to make a hole in' (*hole*); 'to sink a (golf) ball' (*hole*). Call this list *A*. The other access gives the list 'past tense' (-(*e*)*d*); 'past participle, passive' (-(*e*)*d*). Call this list *B*.[12] Details aside, we have enough here in the different behaviour of həʊld and ɑj to prove the point that the phonological forms of words are not all of one and the same type. We must therefore reject the idea that human language is 𝔥(2).

Now consider what it would mean if human language were 𝔥(3). In such a system there are two layers of hash keys, and from the discussion of these systems in the sections above we know that the secondary hash keys are incapable of existing without a primary hash key. For natural language that would mean that there should be phonological forms of words which consist of two parts—the primary hash key and the secondary hash key, where the secondary hash key is an object which has no independent existence, in that it cannot exist on its own as a citation form. But this is exactly the state of affairs we encountered in the behaviour of the 'split' access using həʊld (*holed*). The objects in list *B* are accessible through the 'suffix' d (-(*e*)*d*); and the definitive property of an 'affix' is that it is a 'bound form', namely it is '[a] linguistic form which is never spoken alone' (Bloomfield 1933:160). So equating bound forms with secondary hash keys and 'free forms' with primary hash keys establishes that human language is at least consistent with an 𝔥(3) access system.[13]

This still leaves the option that human language could be 𝔥(4), or higher. If human language were of order 4, then we should expect to find special bound forms (the tertiary hash keys) which cannot occur without a secondary hash key. The only plausible candidates might be 'clitics'. These are bound forms, and there is a considerable literature devoted to disentangling them in principle from 'affixes' (for example, Anderson 1992, Klavans 1985, Zwicky 1977, Zwicky & Pullum 1983). However, it is clear that at the level of the phonological form of these objects, there is *no* difference between them and any other bound form (*i.e.* affixes). Crucially, the distinctions between clitics and affixes that are claimed to exist are given as morphological and syntactic. Further, no-one, to my knowledge, has claimed (or would wish to claim) that there exists a phonological form which can be a possible affix, but which cannot in general be the phonological form of a clitic. Contrast this with the distinction between affixes and free forms discussed in the previous paragraph. The distinction here is *precisely* phonological, and it is most certainly claimed that there exist phonological forms which are possible free forms, but which cannot in general be bound forms.

Since the claim that human language is $\mathfrak{H}(4)$ is severely undermotivated (due to the fact that there appears to be only one phonologically motivated type of bound form), it follows by induction that for any n greater than 3, human language is not $\mathfrak{H}(n)$.[14] Consequently we can be confident in the result that human language uses an $\mathfrak{H}(3)$ access system, where the primary hash keys are free forms, and the secondary hash keys are bound forms (affixes/clitics).[15]

* * *

In the next section we return to the question we earlier postponed, namely the existence of other interfaces than the phonology.

1.5 *Are There Other Access Interfaces?*

It was mentioned in the above discussion that one might plausibly argue that human language makes use of other key systems to access the lexicon. One example might be a system of semantic, or syntactic keys that allows a speaker, during utterance production, to find the appropriate phonological forms to communicate some particular piece of syntactic or semantic structure. Let us call this the *multi-interface hypothesis*.

We can demonstrate quite straightforwardly that it follows from the general properties of hashing mechanisms that the multi-interface hypothesis contradicts *l'arbitraire du signe* (that is, it is a consequence of the multi-interface hypothesis that phonological forms can and must be predictable from semantic forms, and *vice versa*).

An interface is defined by a hashing function, such that for the LObject located at address a, there is a hash key $h=H(a)$. If phonological representations are the hash keys we have $h_\phi=H_\phi(a)$, that is, the phonological object is predictable from the address of the record (the LNode), and *vice versa*. *Mutatis mutandis,* if semantic representations are hash keys we have $h_\sigma=H_\sigma(a)$, that is, the semantic object is predictable from the address of the record (the LNode), and *vice versa*. Since we have $a=H_\phi^{-1}(h_\phi)$ and $a=H_\sigma^{-1}(h_\sigma)$, we also have $h_\sigma=H_\sigma(H_\phi^{-1}(h_\phi))$ and $h_\phi=H_\phi(H_\sigma^{-1}(h_\sigma))$; that is, the relationship between phonological representations and semantic representations is *necessarily* predictable. This means it is not possible for such a system to store idiosyncratic (*i.e.* unpredictable) information. And in the case of natural language, that is false.

Thus, the assumption that some piece of syntactic/semantic structure (a *record*, in the terminology of this section) can already be present in the 'computational system' (the 'syntax'), prior to any phonological information being associated with it must be false. Given that syntactic/semantic information is stored in the lexicon, 'online' syntactic/semantic objects must have already been retrieved from the lexicon. We have only *one* access interface to the lexicon. And this access must be achieved through the key system provided by this one interface, namely, phonological forms. In other words, syntactic/semantic information cannot get into the 'syntax' without *prior* knowledge of the corresponding phonological forms.

While it is logically absurd to maintain *l'arbitraire du signe* and have two sets of hash keys that access LObjects directly, it is perfectly plausible (indeed probably necessary) to assume that there are other lookup tables whose records are (phonological) hash keys. For example, we can imagine that there is a cognitive module which manipulates, say, conceptual-semantic representations. If these representations are to be incorporated into a linguistic structure (and ultimately communicated to other human beings) then that module must have some device which can assign phonological keys to its structures. We may speculate that this is itself achieved through an *n*-order hashing system, with its own separate address space, where phonological keys are the records of the hash system, and conceptual-semantic representations are the keys (opening the possibility of lines of inquiry into 'free forms' and 'bound forms' in cognitive structures other than the phonology). Once this key has been provided, the appropriate syntactic necessary for linguistic processing becomes available from the lexicon.

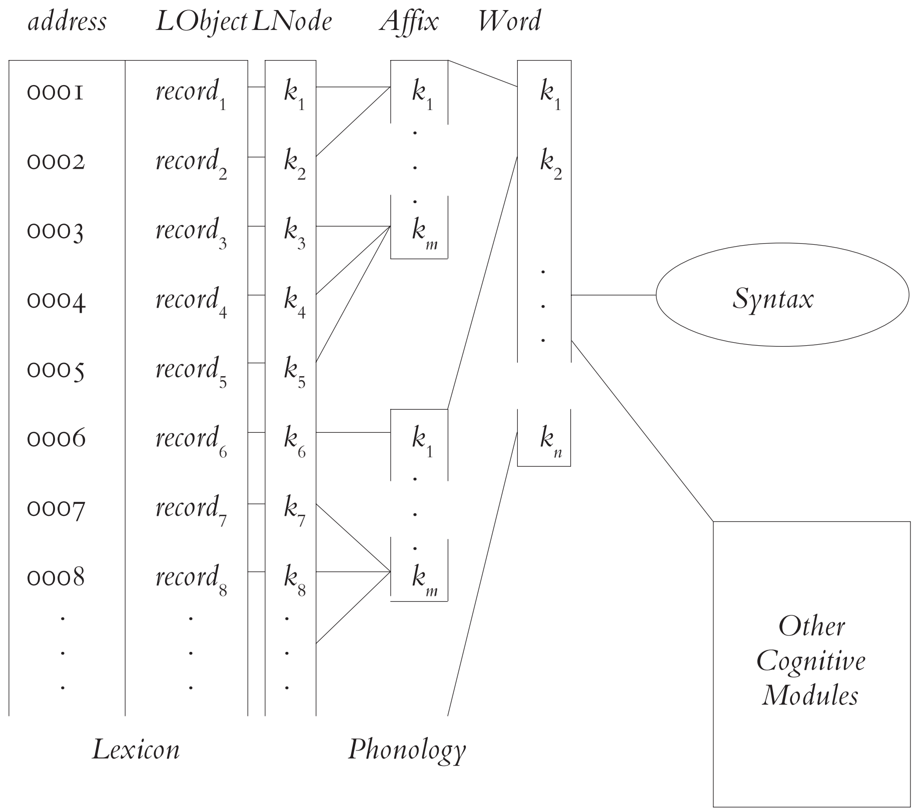
We pursue some of the consequences of this result in the final section (§1.6).

1.6 *Some Consequences*

To recapitulate: the lexicon is a database whose records are LObjects (syntactic/semantic specifications), accessed by an $\mathfrak{H}(3)$ access mechanism (the phonology; primary hash keys being 'free forms', secondary hash keys being 'bound forms'). The 'syntax' is the device which processes the

LObjects. Other cognitive mechanisms can interact with the linguistic system in so far as they can provide access keys (phonological forms) independently of the lexicon (7).

(7) ***The Human Language*** $\mathfrak{H}(3)$ ***System***



One interesting consequence of this position is that we should expect to find that when interfacing with the linguistic system, objects from other cognitive faculties should display exactly the same 'homophony' as we saw with English ɑj. If the only way for, say, the visual system to get one of its objects (say, a picture of an eye) into the linguistic system is through a phonological form (say, that realised as ɑj in English), then we should expect that this visual object would, in principle, make any of the LObjects accessed by ɑj available to the linguistic system (namely, *I, eye, i, aye, etc.*). But this is exactly what we do find, and it has been exploited by many cultures throughout history in *rebus* writing. The phenomenon of rebus writing is commonplace in literate societies, and many writing systems have long been known to have evolved precisely because of such 'visual homophony' (most famously, *Ancient Egyptian*, Gardiner 1957, Loprieno 1995; *Chinese*, Karlgren 1940; *Maya*, Eric & Thompson 1972, Gates 1931).

Next consider a cognitive faculty that manipulates 'conceptual' structures (derived, say, from one of our five sensory functions); let us take *aromas*, for the sake of argument. Now, imagine having smelled a glass of wine, we wish to communicate that we were particularly struck by the vanilla aroma. Our olfactory systems have conspired, we assume, to create a cognitive structure (of which we are aware) corresponding to the particular sensations triggered by vanilla. Let us call this structure VANILLA$_{smell}$.[16] In order to communicate VANILLA$_{smell}$ we need to find a corresponding linguistic object, namely an appropriate LObject. As we have seen in this study, this must be achieved through a phonological key (we have to find a 'word' for VANILLA$_{smell}$). So we need to assume, as mentioned above, that the module which manipulates VANILLA$_{smell}$-type objects has a kind of lexicon, where it can use VANILLA$_{smell}$-type objects as keys to a database containing phonological keys. Let us call an extra-linguistic lexicon like this a *thesaurus*. We are concerned, then with the particular thesaurus which uses VANILLA$_{smell}$-type objects as keys. We call it the thesaurus$_{smell}$. Since a thesaurus is a database, we can expect it to have database properties; we may even expect it to use hashing to organise its data. If that were the case, we should expect to find that the keys of a thesaurus ('concepts') should access, in general, one or more phonological keys ('words'). That this is so is plausibly confirmed by the well known phenomenon of synonymy; thus for wine-drinkers the smell VANILLA$_{smell}$ accesses a list of at least two phonological keys, realised in English as (the phonological forms of) *vanilla* and *oak*. A slightly more familiar example might be DOG$_{sight}$, which might give access to tens of phonological keys (*dog, hound, cur, Fido, Rex, Bonzo, Towzer…*).

If we assume further, and not unreasonably, that the LObject accessed by phonological key *Bonzo* contains, in addition to its morphosyntactic specification, pointers to other cognitive structures which give *Bonzo*, some sort of 'meaning' (that is, a list containing such things as [DOG$_{sight}$,DOG$_{sound}$,DOG$_{smell}$,DOG$_{touch}$,DOG$_{taste}$]), then once DOG$_{sight}$ becomes available in this way, it can be used to access the thesaurus$_{sight}$, which in turn will return the list (*dog, hound, cur, Fido, Rex, Bonzo, Towzer…*). That is, the word *Bonzo* automatically makes potentially available all the other words accessible through DOG$_{sight}$ (in general, through DOG$_{n}$). This sort of 'word-association' is of course a thoroughly familiar phenomenon.[17]

All things being equal, this would make the rather remarkable prediction that the same pieces of brain should 'light up' when processing a visual stimulus to find a word for it as when searching for associated words from a *linguistic* stimulus (and in the absence of a visual stimulus). Unfortunately I do not know of any studies, one way or the other (*cf.* references in footnote 15 to recent functional magnetic resonance imaging (FMRI) studies).

A further interesting question is whether any of the thesauri are $\mathfrak{H}(3)$ or higher. Recall that an $\mathfrak{H}(3)$ system (like the phonology) should display a bound-form~free-form distinction. Sadly my knowledge of other cognitive faculties is not up to the task of answering this question, but I would not consider it beyond hope to await the announcement of the discovery, in the not too distant future, that the structure of our visual cognition, say, utilises a handful of recurrent, dependent structures, which we as linguists would instantly recognise as affixes.

1.7 *Conclusion*

The simple assumptions that formed the foundation of this chapter have produced a somewhat unexpected model of the human language lexicon, and the way it interfaces with other cognitive modules. This model suggests many new lines of enquiry that might fruitfully be undertaken under various disciplinary umbrellas. The speculations of the last section would fall quite naturally into the domain of cognitive neuroscience, while the results of §§1.1–1.4 have significant ramifications for theoretical linguistics (many of which are tackled in the remainder of this work). The detailed account of the computational properties of hashing mechanisms in §1.3 and Appendix A, and in particular their biological instantiation, provides a framework for empirical research in domains such as neurophysiology, and perhaps neurobiology.

I take this variety of domains of potential falsification as an indicator that the theory presented here is useful, and the non-obviousness of its results as an indicator that it may also be insightful.

1.8 *Notes to Chapter One*

1. From *The 6th Exercise: Monday, 3 July* 1922, in Spiller, J. (*ed.*), 1961, *Paul Klee Notebooks, Volume 1, The Thinking Eye*, Lund Humphries, page 449.
2. We note the single dissenting voice of Kaye 1995a.
3. See, for example, Penrose 1995 (in particular Chapter 7) which discusses not only neuronal resources (which are themselves staggeringly huge) but also recent claims made about "microbiological computations" which put significant computational resources at the disposal of each *individual* cell. Penrose notes that at the neuronal level there are computational resources equivalent to a 10^{14} instructions-per-second processor, and at the "microtubular" level this is increased to resources equivalent to a 10^{27} instructions-per-second processor (*op. cit.* p.366).
4. Other 'small lexicon' positions exist, in particular amongst morphologists (Anderson 1992, Beard 1996), which seek to limit not so much the amount of linguistic information

stored, but the number of distinct lexical entries. Both positions, though often set against each other (Lieber 1992), seem to be flip sides of the same 'small is beautiful' coin.
5. It should go without saying that it is both a logical and methodological error to cite as *empirical* evidence mechanisms attributed to the grammar (such as 'rules') which are themselves motivated by the assumption that resources are scarce. Citing apparently rule-based behaviour as corroborating evidence is particularly vulnerable to these errors.
6. Indeed, one's day-to-day experience would seem to indicate that the opposite is true. Humans do seem to go through life happily acquiring new words and taking great pleasure in learning more and more idioms. Further, multi-lingualism is probably the norm in most human societies (so there is enough 'brain space' devoted to language to accommodate several languages simultaneously), and even mature adults have little problem in becoming proficient in new languages.
7. A notable exception is a paper given by Jonathan Kaye and Jean-Roger Vergnaud, which raises the question of the role of lexical access in the organisation of grammar (Kaye & Vergnaud 1990).
8. Note the modularity of this system. The three components (database, computational system and interface) are logically independent in the sense that each component is not 'aware' of the internal structure of any other component. For example, if I wish to send a postcard to Sam in (2) to say 'wish you were here' (*i.e.* I need to access the record 'Sam' in order to 'process it' (send it a greeting)) I do not need to know how the Royal Mail *actually* finds Sam. All I need to know is that Sam's postal address (her 'key') is '3 High Street'. Similarly, the Royal Mail doesn't care what I 'do' with Sam once they have accessed her for me. Nor does the Royal Mail need to know how the local council decides where and when to build houses. It just needs to be able to assign a postal address to each new house as and when it is built. Again, the Royal Mail doesn't need to know who or what is actually located at a postal address. Its job is simply to access the address. Nor does the council need to worry about how the Royal Mail assigns postal addresses. The council's job is to house people. And finally I (the 'data processor') simply want to interact with ('process') my friends, not help them find a house or arrange a postcode for them.
9. That is, the absolute value of the required computational resources never exceeds some constant multiple of n. Informal proof: Assume the interface allocates an individual processor to each key, where each processor can perform its basic operation in (constant) time $T(1)$. Assume further that each processor takes up (constant) $S(1)$ amount of 'space'. Accessing a whole database of size n in one go (*i.e.* in time $T(1)$) requires the allocation of one processor to each of n keys, which in turn requires spatial resources of $S(n)$, which is $O(n)$.

In the general case, assume that there are p processors responsible for each of the n keys, where $p \leq 1$ (one processor can look after one or more keys). The temporal resources in this case increase to $T(1/p)$, and the spatial resources decrease to $S(np)$. That is, the temporal resources required are $O(1)$, and the spatial resources required are $O(n)$. The total resources required for this type of access are therefore always $C + O(n)$, for some (positive) constant C.
10. Note that the logical independence of the modules in (3), as discussed in footnote 2, is not at all compromised by these considerations. The allocation of keys such that they 'pack' the database is a mechanism that is internal to the interface (it is the interface which is responsible for the storage and retrieval of records—it is the Royal Mail which is responsible for issuing postal addresses in such a way that it can effectively deliver the post). By starting with an empty (infinite) database, and letting it grow in size by the addition of finite numbers of records, at any finite time t in the lifetime of the database there will always be some empty key p.

11. The details are left to Appendix A as they do not add anything to the argument, and are quite involved. However, the theorems in Appendix A do provide powerful tools for determining the size and distribution of computational resources in hashing mechanisms. Insofar as the model of the human linguistic system presented in this work is successful, we have the foundation for an investigative programme into the relationships between the actual spatial and temporal resources deployed in the physical brain for the lexicon and its access.
12. These two lists seem to be accessed simultaneously, since only objects which are a legitimate combination of a member of *A* and a member of *B* are accepted as the targets of the access using the whole complex həʊld. In this case the possible targets are 'to make a hole in' + 'past tense' (*holed*); 'to make a hole in' + 'past participle, passive' (*holed*); 'to sink a (golf) ball' + 'past tense' (*holed*); 'to sink a (golf) ball' + 'past participle, passive' (*holed*). The details of these 'split' patterns of access are discussed in Chapter Three.
13. Again, the precise details of this equation are not relevant here, but are treated fully in Chapter Two.
14. From the definition of an $\mathfrak{H}(n)$ system, it follows that for any $\mathfrak{H}(n)$ system there exists an $\mathfrak{H}(n-1)$ subsystem. Thus for any n greater than 4, there must at some point be an $\mathfrak{H}(4)$ subsystem. But we have established that for human language, there is no such $\mathfrak{H}(4)$ system. Therefore human language cannot be an $\mathfrak{H}(n)$ system, where $n>4$.
15. We leave unanswered here two questions that this architecture raises. Firstly, establishing that phonological forms are hash keys entails that theories of phonology need to pay particular attention to constraints on possible phonological free-forms and bound-forms. Hashing mechanisms have associated with them constant properties, and these properties are in principle measurable as physical temporal and spatial quantities (computational resources) of a physical mechanism (like the brain) that instantiates the hashing mechanism. Phonological theories should therefore be designed to be able to deliver empirical statements about these physical constants. Recent developments in neuroscience (*e.g.* Binder *et al.* 1997, Kim *et al.* 1996) provide some optimism that measurement of spatial and temporal resource allocation in the brain during linguistic processing is feasible.

The second unanswered question is that of recursively applied bound-forms (as, for example, *lovingly*). The precise details of the analysis do not change the arguments of this section: there is only one phonologically significant type of bound form in human languages. The recursion of bound forms can therefore be seen simply as a recursion of lexical accesses. Hence 'look up *love* (primary hash key); from this mini-database access the secondary hash key *ing*; from the same mini-database access the secondary hash key *ly*'. We thus have a total of three accesses.

For a thorough discussion of both questions see Chapters Two and Three.
16. We assume that other modalities manipulate analogous structures, insofar as vanilla impinges on them and they have found it necessary or interesting to note them, thus: VANILLA$_{sight}$ VANILLA$_{taste}$ VANILLA$_{sound}$ VANILLA$_{touch}$ *etc.*
17. Of course we can choose to traverse the list of LObjects accessed by any of the phonological keys made available in this way, accessing the verb *to dog*, for example. This in turn will have a list of conceptual structures, which in turn can be used as keys into the appropriate thesauri, and a whole new area of association, to do with doggedly pursuing, becomes available. And so the process continues, constrained, one may speculate, only by the 'relevance' of the results. *cf.* Sperber & Wilson 1986 for ideas about relevance as a constraint on cognitive processing.

Chapter Two

Phonology

夜永酒蘭論及音韵
— *Guangyun*[1]

The results of Chapter One force us to reconsider the role of phonology in the human linguistic system. Because of this change of perspective, many assumptions taken for granted in much of today's phonological discussions are thrown into a different light. Equating the phonology with an arithmetic structure (a hashtable) makes the idea of derivation unobvious; divorcing phonology completely from acoustic (and other) interpretive devices makes the ontology of phonological primitives unobvious.

These and related questions are carefully studied in this Chapter, and we discover that they open a veritable can of worms. The answers are devastating to current theoretical preoccupations, yet the theory that emerges from the devastation is surprisingly simple and surprisingly insightful. And of course it has the benefit of being created primarily as a hashing system.

In §§2.1–2.2 we rehearse the accepted metatheory surrounding the notions of attestation and grammaticality, deriving the theorem that there is no possibility of an interesting definition of grammaticality. In §2.3 we develop a metatheory which highlights those facets of a theory of attestation which can be exploited in various ways, which is done in the following two sections (§§2.4-2.5).

In §§2.6-7 we consider the acoustic interpretation function and the problems of extracting phonological objects from the acoustic environment.

Finally in §2.8 we illustrate all the concepts introduced in this chapter by tracing the early steps of the acquisitional and analytical development of the phonology of the author's native London English, and note some encouraging supporting empirical evidence.

2.1 *Traditional Concerns*

Traditional phonological concerns do not coincide conveniently with the concerns we identified in Chapter One as properly belonging to the phonology. In particular, we have made a complete separation of phonology from any of its interpretive devices, and have come to the conclusion that the only role phonology plays is to define the elements of a hashing system.

The metatheoretical influence of phonetic interpretation on theories of phonology can be seen to be diminishing in a significant, but marginalised, body of research (§2.1.1). The fundamental, perhaps even definitive, traditional concern, however, is the notion of a *possible phonological system* (§2.1.4), which, since Saussure's time, has been formulated along two basic axes: the categorical (§2.1.3) and the linear (§2.1.2).

We accept that this fundamental Saussurean dichotomy is axiomatic (§2.5). However, we believe, and indeed prove in §2.2, that the theories generally proposed to define or derive possible phonological systems suffer from such serious logical flaws that they are best abandoned.

2.1.1 *Phonetics*

Some of the earliest work in phonology, backed up by countless psycho-acoustic experiments, flatly rejected the idea that phonology was determined by articulation. This position was the shibboleth of the European tradition, as represented by the likes of Jakobson and Delattre (Delattre 1968, Jakobson & Waugh 1979), and can be traced back through the Prague Circle (*Travaux du Cercle Linguistique de Prague* 1929–39) to Saussure (Jakobson & Waugh 1979:64):

> L'impression produite sur l'oreille est la base naturelle de toute théorie. La donnée acoustique existe déjà inconsciemment lorsqu'on aborde les unités phonologiques; c'est par l'oreille que nous savons ce que c'est q'un *b*, un *t*, etc.
>
> — *Saussure* 1916, *Cours de linguistique générale.*

It came to more widespread prominence briefly with Jakobson, Fant and Halle's seminal work (Jakobson, Fant & Halle 1952), but has since retreated. Mainstream wisdom is now avowedly articulatory:

> [A]ll functional feature groupings have an anatomical basis.
>
> — *Halle* 1995:2

The only research to abandon the idea that phonology is 'grounded' in phonetics is the *Government Phonology* programme, a position championed most recently in work by Kaye and Harris & Lindsay (Harris & Lindsay 1995, Kaye 1995a). Support for this long-unfashionable position is, however, beginning to be conceded even from the heart of the mainstream itself:

> The term *articulatory* is too narrow in that it suggests that the language faculty is modality-specific, with a special relation to vocal organs. Work of the past years in sign language undermines this traditional assumption.
>
> — *Chomsky* 1995:10 *footnote* 3.

The idea that phonology has a far 'deeper' role, being some sort of addressing system for the lexicon, was expressed as early ago as 1989 (Kaye 1989; Kaye & Vergnaud 1990), and has helped shape several subsequent contributions (Cobb 1995; Jensen 1993, 1995b; Kaye 1989, 1995a).

2.1.2 *Order*

Any theory of phonological representations needs to express *order*. In the past this has been achieved through the device of a *skeleton*, a common construct (in various guises) from the earliest days of autosegmental phonology (Clements & Keyser 1983; McCarthy 1979). In its simplest form, a skeleton is typically a set of *points* structured by a strict linear ordering. The ordering of phonological expressions is achieved by associating them to particular points in the skeleton (Lowenstamm & Kaye 1986; Kaye 1989).

However, such a system does introduce significant complication. Having posited distinct formal systems for expressing order (a skeleton), and phonological categories (a theory of phonological expressions, or a feature geometry) one must ensure that there is also a theory of the relation or relations that hold between the two (association, prosody). This latter part of the formal triple is used to express well-formedness conditions on phonological representations by ruling out certain configurations of phonological expressions and skeleta completely (the licensing principle, government, licensing, distributional constraints).

In the days before the skeleton, order was expressed implicitly in some *string* of symbols, usually *phonemes* (*e.g.* Bloomfield 1933), later *feature bundles* corresponding to *segments* (pioneered in Trubetzkoy 1929 and culminating in Chomsky & Halle 1968). This 'linear' system had the merit of being formally less complex, but was found completely inadequate to the task of ruling out apparently unattested phonological phenomena in any non arbitrary way (Chomsky & Halle 1968:400).

In the theory presented here we take the best of both worlds. We propose *not* to have a separate formal system to represent order, but to represent order *implicitly* through a *string-based* theory (§2.5). However, we agree completely with 'non-linear' theories, and in particular the *non-segmental* ones (Jensen 1994; Kaye 1989) that the 'segment' is *not* a unit over which phonological generalisations are stated (§2.6). That is, we view some of the arbitrariness of the linear theories to be a by-product of the choice of objects subject to the linear ordering, and not as an inherent property of the linearity *per se*.

By making a judicious choice of objects subject to the linear order, we claim that the added complications of autosegmental representations can be abandoned. In §§2.3–4 we spend some time developing and motivating the details of this radical position.

2.1.3 *Categories*

It is an irreducible fact that positions in a given string (or points in a given skeleton) have, or have associated with them, properties other than their position in the linear order which allow them to be differentiated from one another. For example, any theory of phonology must be able to express, minimally, that *dog* and *bog* are *not* identical. The history of phonology is, of course, in large part the story of how best to represent these properties.

The simplest, and most extreme, view is one which attempts to represent all these properties with objects of a single type. This is characteristic of feature-based phonologies (classic examples being Halle 1962; Chomsky & Halle 1968, originating in the work of the Prague Circle: Trubetzkoy 1929). In such frameworks, the objects that are subject to the linear order are called 'segments', and are defined by a bundle of acoustic and/or articulatory features. In general it is assumed that there is a fairly straightforward mapping between these phonological segments and the articulatory-acoustic segmentations given by phoneticians.

However, as discussed at length by Chomsky and Halle (Chomsky & Halle 1968:400*ff*), such theories are unable to rule out a great many apparently unattested phenomena. Their solution was to enrich the theory by endowing features with 'intrinsic content' (*ibid.* p.400), a move that has ultimately resulted in feature bundles being replaced by feature geometries (Clements 1985 up to Halle 1995 and even Harris & Lindsay 1995), or other 'subsegmental' structures (Anderson & Durand 1986; Kaye, Lowenstamm & Vergnaud 1985). Nevertheless, the segments defined by these enriched feature structures are still taken to be the same, phonetically based, ones, and proponents of these theories have expended a great deal of effort on trying to explain why certain strings of these segments are rarely found in human languages.

This endeavour has led to an increasing heterogeneity in the types of objects used to characterise positions in the string. In addition to the featural content of positions, most modern theories admit some sort of 'suprasegmental' theory which captures generalisations about the distribution of features within strings (Brockhaus 1995; Charette 1991; Clements &

Keyser 1983; Goldsmith 1990; Harris 1992; Kaye, Lowenstamm & Vergnaud 1990; McCarthy 1979, *inter alia*).

However, as noted in §2.1.2 above, heterogeneity has the unwelcome side-effect of complicating the overall phonological theory, as subtheories are required to define not only the well-formedness of each new class of category, but also the well-formedness of its relations to each of the other classes of category.

The theory presented here advocates a return to extreme categorial simplicity in the phonology. If we go back to Chomsky and Halle's page 400 dilemma, we note that an alternative logical conclusion is not that the features have 'intrinsic content', but that the choice of features is just wrong. What if the phonetician's segments are *not* in general the objects subject to the phonological linear order? Given our position that the phonology is insulated from any of its interpretive modules, this is a natural conclusion.

Consider the English word *strop*. A typical segmental analysis of this form would comprise five segments, strɔpʰ (more sophisticated theories, like Government Phonology, actually claim seven positions /0strɔp0/, with empty nuclei **0** at the domain edges. Kaye 1992). But why do we not also find fprɔp (or /0fprɔp0/) or the like? As the title of Kaye's 1992 paper suggests, one is forced to resort to endowing the segment s with 'magical' properties. However, freed from the straitjacket of strict segmentalism, we have no reason to suppose that the phonetician's sequence st corresponds to an analogous *phonological* sequence. We could simply claim that st is the (English) realisation of some *single* phonological 'segment' (which in other languages might be realised as some other 'fancy' variety of t).

Consequently, however we decide to define our phonological 'segments', they need to be capable of being 'larger' than the segments we can identify as phoneticians. This strategy is pursued explicitly in §§2.6–7, where a theory of rudimentary phonetic interpretation is given.

2.1.4 *Derivations, Rules and Constraints*

Why do speakers of English not accept aʔnk as an 'English word'? It is generally assumed by phonologists that gaps such as this are 'ungrammatical forms', and are symptomatic of one or more phonological rules (constraints, principles) in the grammar of a speaker being 'broken', or they are symptomatic of non-optimal or disharmonic derivations in the grammar of a speaker (Goldsmith 1993b, Prince & Smolensky 1993).

We could dub this question the *could*-test: "*Could* aʔnk be a word in your language?". The fact that subjects are able to answer this question fairly consistently is taken as evidence of the psychological reality of rules/

derivations/constraints. Consequently the *could*-test is a long-established part of phonological investigation, and is so basic that it is rarely scrutinised.

However, the *could* element of the test is actually inviting the subject's *opinion*. That is, the test is not directly testing the subject's linguistic competence at all, merely their opinions *about* linguistic structures. Given that we know next to nothing about how we form opinions about linguistic structures, and given the anecdotal knowledge that we do have—namely that such opinions are formed largely prejudicially on the basis of what is deemed to be 'good' or 'correct' use of language—any claims made based on data resulting from *could*-tests need to be treated with circumspection.

In §2.3 we provide an alternative interpretation of the *could*-test phenomenon, so that when we are forced by logic to abandon the idea of rules/derivations/constraints (§2.2) we can do so without qualms.

2.2 *The Nature of Grammaticality*

As phonologists, we hold two assumptions which are apparently well motivated, and in a commonly understood sense, 'obvious'. The first of these is the *grammaticality hypothesis*. The grammaticality hypothesis states the belief that there exists a faculty in the human mind that is able to take any well-formed phonological form as input and state whether that form is *grammatical* or not. The second assumption I shall call *attestation*. Attestation is the belief that phonological forms can 'mean something'; that there are no restrictions on what a phonological form can mean; and that any word in any language can become attested for a speaker of any other language. This notion goes back at least to Saussure's *l'arbitraire du signe*. In this section we show that any theory which holds both of these assumptions to be true is *ipso facto* untestable, and hence of doubtful scientific worth.

2.2.1 *Attestation*

Firstly, is there some fairly uncontroversial, empirically verifiable, behaviour that we can observe? The simplest test, I suspect, is one where a subject is asked "what does form p mean?" If the subject replies "it means x" then we can say that form p is *attested*. If the subject replies "I don't know", then we can say that the form is *unattested* :

Definition (2.0). Attestation

A phonological key p is *attested* if and only if there is a (non-null) LObject l such that p accesses l. Otherwise p is *unattested*.

Attestation is a much weaker concept than grammaticality. Thus for the author the form blɪkʰ *blick* is unattested, and so is bʃʧʰ *bshch*. For many English speakers ˈpʰalɪmpsɛst *palimpsest* is not attested, although it might be for anyone interested in papyrology.

Let $\mathscr{A}$ stand for the theory of attestation, and let $\mathbf{\Phi}$ be the set of all possible phonological representations. The theory of attestation is simply a statement of which subsets of $\mathbf{\Phi}$ are possible systems of attestation. To $\mathscr{A}$ we add the following metatheoretical postulate, due ultimately to Saussure, and universally accepted by linguists:

Postulate (2.1). Linguistic Arbitrariness

A new system of attestation can be created from an existing system of attestation by arbitrarily adding new phonological forms to the system.

Formally, *if* $\boldsymbol{P}$ *is in* $\mathscr{A}$*, then any* $\mathbf{Q}$ *such that* $\boldsymbol{P} \subseteq \mathbf{Q}$ *is also in* $\mathscr{A}$*.*

An immediate corollary to Postulate (2.1) is that if the empty set is in $\mathscr{A}$, then $\mathscr{A}$ contains all the subsets of $\mathbf{\Phi}$. The proof follows directly from a well-known result of set theory that the empty set is a subset of all sets. But we are forced to assume another metatheoretical postulate, and that is that $\mathscr{A}$ must contain the empty set. If we were to deny this assumption, we would have that all systems of attestation contain at least one phonological form. We would be forced to conclude, in other words, that an infant is born into the world with at least one attested form ('knowing what some word means'). This is most unlikely to be true, so we accept the postulate, given here as Postulate (2.2):

Postulate (2.2).

There is a system of attestation which contains precisely no attested forms.

Formally, $\varnothing$ *is in* $\mathscr{A}$*.*

Thus, from Postulates (2.1) and (2.2) we derive the theorem that the theory of attested forms is coextensive with the set of all subsets of phonological forms:

Theorem (2.3).

Any set of phonological forms is a possible system of attestation.

Formally, $\mathscr{A} = \mathfrak{P}(\mathbf{\Phi})$.

This result states that $\mathscr{A}$ is simply the same as a (universal) theory of phonological representations. We can also demonstrate that the denial of Postulate (2.1) is false: for any unattested form we can find another system where that form is attested, and then introduce it into the first system (loosely, we can learn what any word in any language means).

*　　*　　*

For convenience we also introduce a family of 'meaning functions' m_i. These functions are specific to an individual at a given time (*i*), and these meaning functions provide a morphosyntactic 'meaning' to any given attested phonological form. We note here that there is now considerable debate about how much of this functional behaviour is achieved through computation ('rules') or through lookup ('lexicon'). From the point of view of the metatheory, these distinctions are irrelevant, and we leave the internal structure of the functions undefined.[2] In fact we let the m_i range over all phonological forms, and define a special 'null meaning' (symbolised by **0**), which means 'unattested'. Thus, an attested form is any form ϕ where $m_i(\phi) \neq \mathbf{0}$.

2.2.2 *Grammaticality*

Mathematically, all theories of grammaticality provide characteristic functions ϕ_i (the phonological competences of speakers *i*) of some subset G_i (the grammatical forms) of the universal set of well-formed phonological objects Φ:

DEFINITION (2.4). GRAMMATICALITY

$\phi_i : \Phi \rightarrow \{grammatical, ungrammatical\}$.

$G_i =_{df} \{p \in \Phi \mid \phi_i(p) = grammatical\}$.

Grammaticality is defined by (the phonological component of) a generative grammar, ϕ_i, of some sort. The common property of all these devices is that they state generalisations of the sort 'any form *p* with property *q* is grammatical'. Without loss of generality, let us call such statements *phonological rules* if the form *p* is a phonological form, and the property *q* is a phonologically defined property. Now it is a commonplace observation that a great many of the phonological rules (in the sense just defined) proposed by phonologists render ungrammatical a number of attested forms. For example the form elma, *elma* (apple), is certainly attested in Turkish, but the textbook account of grammatical Turkish forms requires this form to be ungrammatical, because it is 'disharmonic'. This is not generally considered a serious problem, and a form like *elma* is assigned some idiosyncratic label like 'disharmonic' and is consigned to 'the lexicon'. That is, it seems to be generally, if tacitly, acknowledged that in principle the structure of a phonological rule is *not* a generalisation, but 'a generalisation with some exceptions'. Note that there has been no theory proposed that allows one to determine, by inspecting the rule, whether or not it can sanction 'exceptions'. So given that the device of 'exceptions' is freely resorted to in practice, one can only assume that in principle we must expect that any rule might be broken.

This seemingly innocuous 'tweak' receives little, if any, discussion in the literature, yet its logical consequences are far from innocuous. They are worth examining.

Let $\mathscr{G}$ stand for a theory of grammaticality. A theory of grammaticality is the statement of the set of subsets of $\mathbf{\Phi}$ which are possible grammatical systems.

Phonological practice, as we have seen, sanctions a metatheoretical postulate, which we shall call Exceptions. The insight is that a grammar can accommodate arbitrary (often called 'lexical' or 'prespecified', or 'diacritically marked') exceptions to its rules.[3] An otherwise grammatical object may be prespecified as ungrammatical, or an otherwise ungrammatical object may be prespecified as grammatical:

Postulate (2.5). Exceptions

A new system of grammaticality can be created from an existing system of grammaticality by arbitrarily adding or subtracting phonological forms.

Formally, *if* $\boldsymbol{P}$ *is in* $\mathscr{G}$*, then any* $\mathbf{Q}$ *such that* $\boldsymbol{P} \subseteq \mathbf{Q}$ *or* $\mathbf{Q} \subseteq \boldsymbol{P}$ *is in* $\mathscr{G}$.

From Postulate (2.5) it follows immediately that every set of phonological forms is in $\mathscr{G}$. This is proved easily: since all subsets of a set in $\mathscr{G}$ are in $\mathscr{G}$, the empty set must be in $\mathscr{G}$; if the empty set is in $\mathscr{G}$, then all sets must also be in $\mathscr{G}$, since the supersets of all sets in $\mathscr{G}$ are in $\mathscr{G}$, and all sets are the super sets of the empty set. Hence we have the following theorem:

Theorem (2.6).

Any set of phonological forms is a possible system of grammaticality.

Formally, $\mathscr{G} = \mathfrak{P}(\mathbf{\Phi})$, hence $\mathscr{G} = \mathscr{A}$.

That is, the theory of grammaticality is the same as the theory of attestation. This result is surprising for anyone who takes the notion of grammaticality seriously, which is perhaps most phonologists. The stumbling block, of course, is Postulate (2.5). But consider, as a theoretical grammarian, what it would mean to deny Postulate (2.5). It would mean denying that a grammar could accommodate arbitrary exceptions; in other words a grammar would have to be exceptionless. Unfortunately there is no grammar yet proposed which is claimed to be exceptionless, and Postulate (2.5) remains a fact of current phonological practice. In fact Postulate (2.5) is occasionally explicitly asserted, with no apparent inkling of what it entails (recently, for example, Inkelas, Orgun & Zoll 1997). Even when not explicitly stated, it follows that any theory whose metatheory allows the expression of arbitrary exceptions is rendered as uninteresting as Theorem (2.6) indicates.

Phonologists in the generative tradition may object that they have a 'deeper' concept of grammaticality, because of their commitment to the 'psychological reality' of phonological rules. But note that if a system of grammaticality is defined by a set of rules, we can provide counter-examples to any rule simply by constructing a new grammatical system which contains the counter-example as a 'lexically marked exception' (in other words, we can falsify any rule). Conversely, from the grammarian's perspective, any such counter-example we supply can be 'accounted for' by marking it as an exception. That is, *any* proposed system of rules, psychologically real or otherwise, is by definition insulated against empirical testing. This corollary to Postulate (2.5) is so important that we declare it here as Dilemma (2.7).

Dilemma (2.7).

i. Statements of grammaticality given in a system of rules are false.

ii. Statements of grammaticality given in a system of rules and exceptions are untestable.

So an interesting theory must be one for which Postulate (2.5) is false, namely, it is not in general possible to create a new system of grammaticality from an existing system of grammaticality by arbitrarily adding or subtracting forms. To deny Postulate (2.5) is to say that there is some form (in some language) which could never become 'part of one's language'. This seems counter-intuitive: as linguists we are accustomed to believing in the phenomenon of 'loanwords', or 'borrowing', a natural conclusion to draw from our assumptions about linguistic arbitrariness, and corroborated by our day-to-day experience.

Methodological conveniences like the 'idealised speaker/hearer' (*e.g.* Chomsky 1986, going back to Bloomfield 1933) are no solution. They lead to theories which on the one hand license arbitrary selection of 'relevant' data, and the disqualification of awkward data; on the other hand such a methodology seems to me to ignore perhaps one of the most fundamental and pervasive facts of linguistic usage, namely the life-long ability we have to create and adopt novel linguistic forms, *despite* the prescriptions of an idealised speaker/hearer's grammar.

Assume, then, that Postulate (2.5), and hence Dilemma (2.7), is true. The logic forces us to conclude that we shall not find an interesting theory of grammaticality. This leaves us rather impoverished, as systems of grammaticality defined by rules and exceptions are enlisted to account for other facets of linguistic behaviour, besides statements of apparent inventory. The question is whether we can still insightfully investigate these phenomena with tools developed in the stricter metatheory. We maintain that it is indeed possible, and accordingly abandon the notion of grammaticality and its rule-based *accoutrements*.

2.3 *A New Approach*

In the vacuum created by the consignment of grammaticality to history, we consider whether or not it is possible to exploit the attestation hypothesis to develop theories of human phonology. Happily we arrive at an affirmative answer to this question. The following two sections (§§2.3–4) present in considerable detail the requisite metatheory, along with an instructive implementation of a simple phonological theory in which we are able to observe apparent rule-based behaviour, simply by 'measuring' the distribution of attested forms.

One phenomenon we need to address is our ability to process unattested forms (such as *blick*). Because we believe that our theories are about real human beings, we must assume that any human being could potentially instantiate any system of attestation in $\mathscr{A}$. In other words, any human being could instantiate any set of phonological forms. So it seems reasonable to assume that we are capable of processing all phonological forms. Therefore we can utter and parse phonological forms which are not attested for us, simply because we have a phonetic interpretation function (*q.v.* §2.6).[4]

However, the chief area we need to address in the absence of a notion of grammaticality is 'affiliation', or the intuitions speakers have about whether a form 'belongs to their language'.[5,6] We take great pains in §§2.3–2.5 to demonstrate that theories of judgements about affiliation can be developed without the notion of grammaticality, based simply on the distribution of attested forms.

2.3.1 *Meta-phonology*

The sorts of distributional restrictions associated with intuitions about affiliation, so-called *systematic gaps*, continue to occupy the forefront of phonological research, and a wide variety of notations and devices have been proposed to account for them (see for example the nice synopsis in Goldsmith 1995b). However, before plunging headlong into any similar such 'accounting', it would be wise to stick to our sceptical, minimalist principles, which have guided the construction of the present theory thus far.

In direct consequence of our change of perspective, we can view intuitions about linguistic affiliation as information *about* phonological forms, rather than as a system which defines licit and illicit sets of phonological forms. That is, we may view phonological forms as coming with associated information. One component of this information is a judgement about affiliation ("doesn't sound English to me"). Another way to view this is that 'grammaticality judgements' are nothing more than the expression of certain reactions to the distribution of morphosyntactic meanings across phonological forms: that is, reactions to the meaning functions m_i.

Now, it is possible to define a notion of affiliation by considering the inherent structure of the set of all phonological representations. We can define a measure of distance over Φ, and can relate affiliation judgements to these distances. We note that some form of distance metric is required anyway, because we have intuitions about how 'related' phonological forms are. The rich variety of word games (including poetry) which rely solely on relations between phonological forms is a particularly striking manifestation of these intuitions.[7] We accordingly accept the metatheoretical postulate that there is a notion of phonological distance:

POSTULATE (2.8).
There is a notion of distance *between phonological forms.*

The actual definition of this notion of distance will, of course, ultimately depend on the given theory of phonological representations. We undertake such a definition in §2.5.

Now we can approach the problem of affiliation. It seems that affiliation is akin to distance. To say that such-and-such a word 'doesn't sound English', is simply to say that the word 'doesn't resemble (or is a large distance away from) most of what I would call English words'. So all we need is a characterisation of the intuition 'most of what I would call English words'. We can then use our pre-existing distance metric to measure how far any form is from this 'most of ...', and consequently generate statements about how 'English' a form is. What we need, then, is some sort of speaker-dependent 'average' of 'existing' forms:

Postulate (2.9).
There is an individual-specific notion of an average phonological form.

The only individual-specific notion we have to hand is attestation, specifically the functions m_i. Let us therefore metaphorically think of attestation as *mass*—an unattested form has no mass, and an attested form has a mass. Let us further assume that each separate morphosyntactic 'meaning' contributes one unit of mass to any phonological form which attests it (roughly, the more polysemous a form is, the 'heavier' it is). Let us symbolise the mass of a form, ϕ, by 'counting' the number of morphosyntactic representations returned by $m_i(\phi)$. Thus the mass of $\phi =_{df} |m_i(\phi)|$.[8]

Now, with this mechanical analogue in mind a straightforward 'average' suggests itself—the *centre of mass*, or the *barycentre*. Phonological forms are simply points in a phonological space, they have a mass, ($|m_i|$), so we can calculate their centre of mass from an appropriate definition. A form which is a greater distance from this 'centre of gravity' of attested forms should elicit judgements about corresponding degrees of 'weirdness'.

Before pursuing the details of such a system (§2.5), let us reflect briefly on some of the macro-level phenomena we might expect to observe if linguistic systems really are characterised in the way just suggested.

2.3.2 *Macro-phonology*

Given that the barycentre of an individual's system is defined as an average, it follows that it changes with the attestation of any new form, and hence that judgements about affiliation will in principle change. Further, again because the barycentre is an average, it follows that in a large system, the attestation of one or two forms will have only a slight effect on the barycentre. That is, a mature system will be relatively undisturbed by the odd new attestation, whereas an immature system (with very few attested forms) will be affected much more. And interestingly, if there was a sudden influx of many, previously unattested forms (a large new corpus of loanwords, say), then even the barycentre of an already massive system would be perturbed.[9] But note also that the barycentre can still be perturbed by the gradual accumulation of small numbers of new attestations.

Again, the mechanical analogue proves useful: if the ratio of the total mass (the inertia) of the system to the inertia of the 'loanword system' is small, the system will seem very plastic; whereas if the ratio is large, the barycentre will not shift so much.

If this average is indeed the metric of linguistic affiliation we intend it to be, and given that language is one of the most powerful tools defining our social groups, we should expect culture-specific reactions to shifting barycentres. That this is true is possibly a truism. From *l'Académie française* to the dinner-table exhortations of parents, there is no doubt that it is novel attestations which are seen to pose the greatest threat to established linguistic (and hence cultural) identities. This is perhaps because we are sensitive to the barycentric shifts that contact with new linguistic forms brings.

A related area may be language acquisition, where we might assume again that the least disruptive of candidates is the preferred structure for acquisition. It is worth noting that in both acquisition and analysis there are two related, but distinct, states. The first we might call the *learning* state. In this state the L[anguage]A[cquisition]D[device]/analyst accepts that the encountered datum is to be 'learned', that is, integrated into the system. In this state we expect a barycentric shift. The second state we might call *diagnostic*. In this state the LAD/analyst compares the encountered datum with the existing system, and delivers a 'diagnosis' of how well the form fits with the existing system. In this state we do not expect a barycentric shift. As the LAD matures, or as the analyst becomes more confident of a given analysis, the two states almost certainly take on a more integrated role, with diagnosis preceding acquisition. As time goes by, therefore, acquisition will appear to become more and more discriminating, as only those forms with a favourable diagnosis will be accepted for acquisition.[10]

Thus, in the early stages of analysis/acquisition, we are very indiscriminating about encountered forms, tending to accept any as acquirable. This means of course, that a malicious experimenter, or malicious parent, could deliver us many (nonsense) forms, like rgɪr, which could indeed have an effect on the later diagnoses of the system, making forms with, say, initial rg sequences more acceptable than they would be to a system which had not been exposed to the malicious data. A mature system, where diagnosis is a pre-requisite to acquisition, would resist acquiring malicious data which could be diagnosed as too distant from the established barycentre.

This behaviour does seem to be borne out empirically. Consider 'malicious' data to be 'foreign language' data. An immature LAD acquires the 'foreign' data indiscriminately with the 'non-foreign' data, resulting in a range of mature systems with increasing numbers of 'loanwords' in them, with multilingualism, we might surmise, in the limit. A mature LAD, however, tends to resist 'foreign' data precisely because it is diagnosed as barycentrically distant (or 'foreign').

Imagine further that there is significant pressure on the mature LAD to acquire a given 'foreign' datum. The LAD can simply bite the bullet, as it were, acquire the datum, and accept the resulting barycentric shift. Alternatively, we can assume a *normalising* function which can change the foreign datum so that its diagnosis becomes more favourable. Whatever strategy is employed is an individual- and datum-specific choice, conditioned, no doubt, by many external factors.

⋆ ⋆ ⋆

These macro-phonological threads provide ample scope for future research, some of which we defer and some of which will occupy us greatly in §§2.7–8. But we first pursue in the next section the micro-phonological possibilities of a system constrained in accordance with our established meta-phonological postulates.

2.4 *Micro-phonology*

In this section we instantiate the metatheoretical postulates of the previous section with a toy phonological theory. By 'toy' we mean a simplified theory, an idealisation created for the purposes of exposition. However, we shall be very careful to say in what ways our idealisation differs from a more complete implementation, and shall show that our idealisation is really only one of scale. We do not intend that any further metatheoretical postulates are required for an adequate account of phonological behaviour.

2.4.1 *Fundamental Axioms*

We summarise some assumptions that were introduced in the discussion of metaphonological postulates in §2.3 above. These definitions become axioms of our theory.

Definition (2.10).

i. *Phonological forms are points in phonological space*
ii. *Attestation is the distribution of mass in phonological space*
iii. *The average phonological form is the centre of mass (barycentre) of this distribution*

The definitions above need some further discussion and sub-definition. We begin with a review of the notion of centre of mass. Consider two point masses on a plank of wood. The centre of mass is that point on the plank where, if one were to place a fulcrum, the plank would balance. Let the first mass, m_1 be at point x_1 along the plank, and let the second mass, m_2 be at point x_2, then the centre of mass is at point $\hat{x}$:

$$\hat{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

The definition generalises straightforwardly to a system of arbitrary points with arbitrary numbers of co-ordinates (or dimensions):

Defition (2.11). Position of the Barycentre

For a system of n point masses m_n, in a j-dimensional space, where the position vector, $\mathbf{r}_i$, of each m_i is $\mathbf{r}_i=(z^1{}_i, z^2{}_i, z^3{}_i, \ldots z^j{}_i)$, and where $M=m_1+m_2+\ldots m_n$, the position vector of the barycentre $\mathbf{r}_\mu=(z^1, z^2, z^3, \ldots z^j)$ is given by

$$\mathbf{r}_\mu = \frac{1}{M}\sum_{i=1}^{n} m_i \mathbf{r}_i$$

Analogously, we can talk about the centre of mass (or barycentre) of two attested forms, ϕ_1, ϕ_2. The attested forms correspond to the point masses, where the mass, as discussed above, is given by the amount of morphosyntactic 'matter' associated with each form ($\mu_1=|m_i(\phi_1)|$ and $\mu_2=|m_i(\phi_2)|$); the actual phonological representation corresponds to a coordinate in phonological space, or a position along a 'phonological plank'. Using the formalism developed above, we can state the barycentre, ϕ_μ, of this simple two-word language as

$$\phi_\mu = \frac{\mu_1\phi_1 + \mu_2\phi_2}{\mu_1 + \mu_2}$$

Now, all we need to ensure is that our theory of phonological representations is stated in such a way that we can meaningfully talk about the multiplication and division of a phonological form by a real number, which we will call a 'scalar' (we need to multiply and divide by masses. The masses in the above equation are scalars). We also need to talk about distances from the barycentre, so we need a meaningful interpretation of the notion of addition and subtraction of these (scalar × phonological-form) products. By equating phonological forms (in some way) with positions, or co-ordinates, in an appropriately defined space, we can borrow a ready made and long established mathematical tool which allows us to manipulate these quantities: *vectors*.[11]

So let us consider the internal representation of our two phonological forms ϕ_1 and ϕ_2. A theory of phonology needs to define what a possible phonological form is. This is usually achieved through some formal calculus which typically states what the representational primitives are (features, elements, nodes, feet, onsets *etc.*), and a means of combining them (matrices, fusion, association, prosodic hierarchy, licensing *etc. Cf.* §2.1) .

2.4.2 *Toy Phonology* I

Let us design an extraordinarily simple phonological theory, which is, as it were, a theory of our simple 'phonological plank'. We define a set of three representational primitives {**A**,**I**,**U**}, and we define a well-formed phonological form to be any subset of {**A**,**I**,**U**}. Thus our entire phonological repertoire in this simple phonological world consists of maximally eight forms: {{},{**A**},{**I**},{**U**},{**A**,**I**},{**A**,**U**},{**I**,**U**},{**A**,**I**,**U**}}. Call this set **Φ**. This set is the phonological plank. In our two-word language, the forms ϕ_1 and ϕ_2 lie on this plank. Let us say they lie at {**A**} and {**A**,**I**,**U**} respectively, and let us further assume they both have a mass of 1; the total mass of the system, its inertia, is therefore 2.

Calculating the barycentre of this two word language allows us to provide 'affiliation judgements' for any phonological form. We can therefore determine how a speaker of this simple language would react to a new form, say {**I**}. From the equation above we have that the barycentre is at

$$\phi_\mu = \frac{1\times\{\mathbf{A}\} + 1\times\{\mathbf{A},\mathbf{I},\mathbf{U}\}}{2}$$

So the barycentre is located at ½{**A**}+½{**A**,**I**,**U**}. In order to get a handle on how to interpret this quantity, we look to the analogy that phonological representations are points in a space. The barycentre then is a point that is 'halfway to point {**A**} and halfway to point {**A**,**I**,**U**}'. We note that this quantity is not in itself a phonological form as defined by our theory (it is not a point in **Φ**); in terms of the plank analogy, it is as if the phonological forms can only occupy certain discrete positions along the plank (like the notches in a ruler, say), but the barycentre (the point of balance) can be *anywhere* along the plank, even between the notches of the ruler.

Consequently, the set **Φ** is not an adequate characterisation of the space in which phonological forms live. The set **Φ** is actually analogous the notches on the ruler. So we need to define a denser space, the fabric of the ruler, so that we can locate the barycentre in it. And this is where the vector notion becomes useful. With a simple change of interpretation we can transform our simple phonological system into a multi-dimensional vector space with just the continuum properties we require.

Let the primitives of our phonology {**A**,**I**,**U**} be the unit vectors defining the axes of a three-dimensional (Euclidean) space (*fig.* 1). And let phonological representations be the position vectors of apexes of the unit cube. Then it follows that a phonological representation {**A**,**I**} is the vector **A**+**I**, the position vector of apex (1,1,0). Vectors can be multiplied by scalars, so the vector *a***A** is the position vector of the point (*a*,0,0), and similarly the vector *a*(**A**+**I**) is the position vector of the point (*a*,*a*,0).

FIGURE 1

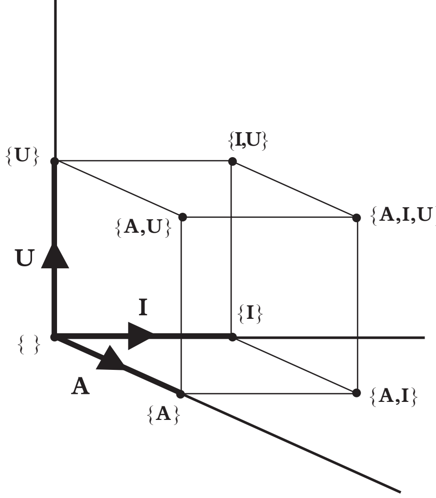
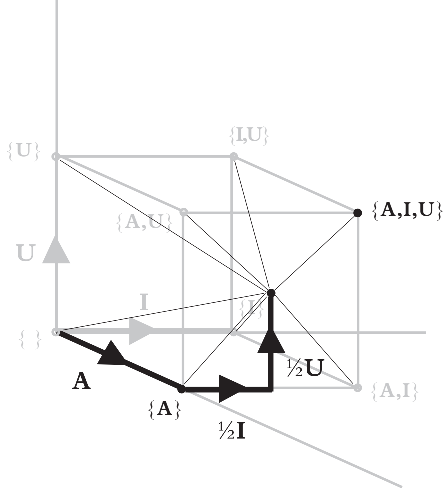


FIGURE 2



Now we can locate the barycentre of our two-word language (*fig.* 2). The position vector of the barycentre is ½**A**+½(**A**+**I**+**U**), which is (**A**+½**I**+½**U**), or point (1,½,½). This point is the average phonological form of the two-word language, and represents the origin from which distances are measured to judge the 'weirdness' of phonological forms.

With phonological space thus transformed into a Euclidean 3-space we have a ready-made definition of distance, familiar in its two-dimensional version as 'Pythagoras's Theorem'. The square of the distance of a point from the origin is simply the sum of the squares of its coordinates. Thus the distance of point {**A**}, with position vector **A**, from the origin (that is, from (0,0,0)) is the square-root of $1^2+0^2+0^2$, which is 1. The point {**A**,**I**} is distance $\sqrt{(1^2+1^2+0^2)}$, $\sqrt{2}$, approximately 1.414. In general the square of the distance between any two points is equal to the sum of the squares of their coordinate differences.

DEFINITION (2.12). DISTANCE METRIC

For any two points $\mathbf{x}=(x^1,x^2,x^3,\ldots x^n)$ *and* $\mathbf{y}=(y^1,y^2,y^3,\ldots y^n)$ *in an n-dimensional Cartesian co-ordinate system, the distance between* **x** *and* **y**, *symbolized* $|\mathbf{x}-\mathbf{y}|$ *is given by*

$$|\mathbf{x}-\mathbf{y}|^2 = \sum_{i=1}^{n}(x^i-y^i)^2$$

So the distance of point {**A**} from the barycentre in our two-word language is $\sqrt{((1-1)^2+(0-½)^2+(0-½)^2)}$, $\sqrt{½}$, approximately 0.707. We can perform this calculation for all possible phonological forms, and so arrive at a complete characterisation of the affiliation judgements of a speaker of our two-word language.

Form	*Point*	*Barycentric distance*	*Form*	*Point*	*Barycentric distance*
{}	(0,0,0)	$\sqrt{1½}$, ≈1.22	{**A**}	(1,0,0)	$\sqrt{½}$, ≈0.707
{**I**}	(0,1,0)	$\sqrt{1½}$, ≈1.22	{**U**}	(0,0,1)	$\sqrt{1½}$, ≈1.22
{**A**,**I**}	(1,1,0)	$\sqrt{½}$, ≈0.707	{**A**,**U**}	(1,0,1)	$\sqrt{½}$, ≈0.707
{**I**,**U**}	(0,1,1)	$\sqrt{1½}$, ≈1.22	{**A**,**I**,**U**}	(1,1,1)	$\sqrt{½}$,≈0.707

An interesting point comes to light on inspection of the above table, and that is that there are phonological forms which are not attested, but which are as close to the barycentre as one or other of the two attested forms. That is, a by-product of the pattern of attestation for our speaker is that certain other forms, although not attested, would in fact not be considered 'as weird as' other non-attested forms, and in fact may even be considered 'as good as' one or other of the two existing forms. Inspecting the above table, we see that {}, {**I**}, {**U**} and {**I**,**U**} are all furthest from the barycentre ('weird'), while the rest, {**A**}, {**A**,**I**}, {**A**,**U**} and {**A**,**I**,**U**} are closest to the barycentre ('not weird').[12]

A helpful auxiliary notion we can introduce here is the *threshold of acceptability*, which we define simply as the greatest attested distance from the barycentre. We can then say that any form whose barycentric distance is outside this threshold is 'weird', and any form whose barycentric distance falls within the threshold is 'non-weird'. The threshold of acceptability defines an *n*-dimensional sphere which contains all non-weird forms:

DEFINITION (2.13). THRESHOLD OF ACCEPTABILITY

The threshold of acceptability ρ, *of a system of i attested forms* $\mathbf{r}_i$ *with barycentre* **R** *is defined as* $\rho =_{df} \max(|\mathbf{r}_i-\mathbf{R}|)$.

Any form **r** *such that* $|\mathbf{r}-\mathbf{R}|\leq\rho$ *is* non-weird. *If* $|\mathbf{r}-\mathbf{R}|>\rho$ *we say* **r** *is* weird.

So, in the example above, we can see that the threshold of acceptability of our simple system is 0.707. Therefore the orbit of acceptability includes all those forms whose barycentric distance is no greater than 0.707.

Let us now see what happens to our speaker when she acquires two new forms, say on exposure to a speaker of the {**A**} and {**I**,**U**} language of footnote 12. Our speaker now has two meanings associated with {**A**}, and a completely new form {**I**,**U**}, which used to be considered quite weird for her. The inertia of the new system is twice that of the old, being now 4. The new barycentre is located at ¼(2**A**+(**A**+**I**+**U**)+(**I**+**U**)) =(¾**A**+½**I**+½**U**), or point (¾,½,½). The new table of our speaker's intuitions now looks like the following:

Form	*Point*	*Barycentric distance*	*Form*	*Point*	*Barycentric distance*
{}	(0,0,0)	≈1.03	{**A**}	(1,0,0)	=0.75
{**I**}	(0,1,0)	≈1.03	{**U**}	(0,0,1)	≈1.03
{**A**,**I**}	(1,1,0)	=0.75	{**A**,**U**}	(1,0,1)	=0.75
{**I**,**U**}	(0,1,1)	≈1.03	{**A**,**I**,**U**}	(1,1,1)	=0.75

We find that the overall 'gut reactions' are the same, but that the previously weird forms are now not weird, since the threshold of acceptability has moved to 1.03, which includes every form in the system. In a similar fashion, by varying inertias, it is possible to prove the macro-phonological claims made in §2.4 about the gross behaviour of systems in contact.

It is worth pausing to reflect that we have achieved these results without a rule component, and without parameter setting, and without constraint ranking. We just measured things known already.

* * *

A real-life theory will involve many degrees of freedom. Firstly there are linear degrees of freedom: a phonological form is not typically just one 'segment' long. Let's say we restrict ourselves to 'phonological words' as the largest phonological entities, and stipulate (perhaps controversially, but *cf.* §2.6) a maximum word length of w 'skeletal points'. Next, each skeletal position is characterised by being dominated by a certain number of 'super-skeletal' objects ('prosodic categories'). Let's say our theory has a total of p possible configurations of prosodic categories that can dominate a point. Next, each skeletal position has 'melody' associated to it. Again, let us say that there are m possible melodic complexes that can attach to a point. So, our phonological forms exist at the apexes of the ($w+p+m$)-dimensional unit hypercube. But once we have made this transformation into the new ($w+p+m$)-dimensional Cartesian co-ordinate system, we can calculate the barycentre using the same methods we used in the toy implementation, from the generalised Euclidean metric given in Definition 2.11.

Before scaling up to a life-sized phonology, we consider one more toy phonology, which introduces linear order into phonological representations. In this way, with the two toy phonologies we shall have illustrated the mechanisms at work (in fact, one and the same mechanism) in the 'categorial' and 'linear' domains discussed in §2.1.

2.4.3 *Toy Phonology* II

In this sub-section we illustrate how the linear order of a phonological form can be subsumed by the barycentric vector method. We extend the toy phonology of the previous discussion and allow that a phonological form consist of exactly two phonological expressions taken from $\mathbf{\Phi}$. Thus, all the phonological forms in this new phonological theory are the members of the set of ordered pairs of elements from $\mathbf{\Phi}$. Call this set $\mathbf{\Phi}^2$. To simplify the presentation further, we introduce the following notational convention for members of $\mathbf{\Phi}$:

$$\mathbf{0} =_{df} \{\ \} \qquad \mathbf{a} =_{df} \{\mathbf{A}\} \qquad \mathbf{i} =_{df} \{\mathbf{I}\} \qquad \mathbf{e} =_{df} \{\mathbf{A},\mathbf{I}\}$$
$$\mathbf{u} =_{df} \{\mathbf{U}\} \qquad \mathbf{o} =_{df} \{\mathbf{A},\mathbf{U}\} \qquad \mathbf{ü} =_{df} \{\mathbf{I},\mathbf{U}\} \qquad \mathbf{ö} =_{df} \{\mathbf{A},\mathbf{I},\mathbf{U}\}$$

In any given phonological form, the linear order can be viewed as a parameter, such that for any given value of the parameter (position in the linear order), there exists a unique phonological expression. Thus the phonological form **oi** can be viewed as a vector function $\boldsymbol{f}_{\mathbf{oi}}(t)$ where $\boldsymbol{f}_{\mathbf{oi}}(1)=\mathbf{o}$ and $\boldsymbol{f}_{\mathbf{oi}}(2)=\mathbf{i}$. That is, **oi** (or more correctly $\boldsymbol{f}_{\mathbf{oi}}$) defines a *curve* in phonological space. We can imagine the mass of some 'body' which follows **oi** to be uniformly distributed along the curve. In our simple system, any curve consists of exactly two points, where $t=1$ and $t=2$, hence a phonological form **oi** with mass m can be viewed as a body following the curve $\boldsymbol{f}_{\mathbf{oi}}$ with a mass distribution $m(t)$ such that $m(1)=½m$, $m(2)=½m$ (in general, for a curve with τ points, the uniform mass distribution is $m(t)=m\tau^{-1}$).

The question of linguistic affiliation posed above thus becomes one of finding the 'average curve' in phonological space, and of finding some measure of divergence between curves. Let the average curve be a vector function $\boldsymbol{f}(t)$. It seems reasonable to assume that the values of this average are simply the average phonological forms for all defined values of t.

DEFINITION (2.14). THE BARYCENTRIC CURVE

For a system of total mass M, with n attested forms, each form i describing a curve $\mathbf{s}_i$, *where there are* τ *points along any curve, we have the total mass distribution* $M(t) = M\tau^{-1}$ *and mass distributions* $m_i(t) = m_i\tau^{-1}$ *and*

$$f(t) = \frac{1}{M(t)} \sum_{i=1}^{n} m_i(t)\mathbf{s}_i(t) \qquad \textit{hence,} \qquad f(t) = \frac{1}{M} \sum_{i=1}^{n} m_i \mathbf{s}_i(t)$$

That is, a *local barycentre* is defined for each value of the parameter *t* (position in the linear order). In our example we have a single form **oi** with mass *m*. Every curve exists at two points, and the inertia of the system is *m*. Hence the average phonological form is (**A**+**U**,**I**), *i.e.* the curve from the point (1,1,0) to the point (0,1,0).

How different is a curve **s** is from another curve **r**? The question involves a little extra work than the differences between points, since curves are geometrical figures, which have shape-like properties such as gradient and 'curl'. In comparing curves, then, we must be careful to include all aspects of the geometry of the curves. For example, it will not do simply to compare values of both curves for each value of *t*. For in general, although two curves may share the same points, they may behave drastically differently in between those two points, to such an extent that despite their similarity for all values of *t* we may not be willing to call them 'similar'.

The usual way of investigating these properties is to analyse the way in which a curve *changes* over *t*. If two curves seem to change in a similar way, as well having similar values at the designated points in *t*, we should be more willing to call them 'similar'. Because our phonological curves (even when scaled up to full-size curves) are *discrete* we do not need to enlist the full power of the differential calculus, but can employ the discrete equivalent: finite differences. Thus we can evaluate the rate of change of a curve as it moves between two consecutive points x and $x+1$ simply by evaluating the *difference* between the value of the curve-function at $x+1$ and x. These differences are known as *differences of the first order*. If the curve is long enough, we need to take account of higher order differences too, as these give us information about the rate of change of the rate of change *etc.* Thus the first second order difference of a curve is given by calculating the difference between the second first order difference and the first first order difference. The values of the curve itself are the 0-th order differences.

DEFINITION (2.15). *n*-TH ORDER DIFFERENCES

For a curve $\boldsymbol{f}$ *with t discrete points, there are exactly t differences of order* 0, $t-1$ *differences of order* 1, $t-2$ *differences of order* 2, … 1 *difference of order* $t-1$, *or, a total of* $\frac{1}{2}(t(t+1))$ *differences.*

The n-th order differences δ_i^n *of* $\boldsymbol{f}$, *where* $1 \leq i \leq t-n$, *are given by*

$$\delta_i^0 \boldsymbol{f} =_{df} \boldsymbol{f}(i);$$
$$\delta_i^n \boldsymbol{f} =_{df} \delta_{i+1}^{n-1} \boldsymbol{f} - \delta_i^{n-1} \boldsymbol{f}$$

The similarity of two curves can be measured straightforwardly by making a pairwise comparison of all corresponding differences of all orders. As these differences approach each other, so the two curves approach each other. We can also define a measure of similarity by summing the (squares of the) magnitudes of the differences between the pairs of corresponding differences.

DEFINITION (2.16). MEASURE OF DIVERGENCE

The measure of divergence $\Delta(\boldsymbol{g}-\boldsymbol{f})$ *between curves* $\boldsymbol{f}$ *and* $\boldsymbol{g}$ *is given by*

$$(\Delta(\boldsymbol{g}-\boldsymbol{f}))^2 = \sum_n \sum_i |\delta_i^n \boldsymbol{g} - \delta_i^n \boldsymbol{f}|^2$$

Finally, we can provide a generalised notion of the threshold of acceptability, by defining it over the measure of divergence.

DEFINITION (2.17). (GENERALISED) THRESHOLD OF ACCEPTABILITY

The threshold of acceptability ρ, *of a system of i attested forms* $\mathbf{r}_i$ *with barycentre* $\mathbf{R}$ *is defined as* $\rho =_{df} \max(\Delta(\mathbf{r}_i - \mathbf{R}))$.

Any form $\mathbf{r}$ *such that* $\Delta(\mathbf{r}-\mathbf{R}) \leq \rho$ *is* non-weird. *If* $\Delta(\mathbf{r}-\mathbf{R}) > \rho$ *we say* $\mathbf{r}$ *is* weird.

One of the consequences of this view, which seems to be borne out empirically, is that 'weirdness' should be locatable at a given point within a phonological string. Most significantly, however, is the corollary that each position in the string has its own mini-barycentre; that is, what may be weird in position 1 may not be weird in position 2, and vice versa.

Again, no rules have been stipulated, we have just measured the distribution of attested mass. This position-dependent 'systematicity' has long been recognised in phonological investigations. Consequently it makes little sense to talk in overall terms of a speaker's 'inventory'. We should rather talk in terms of apparent 'inventories' with respect to given positions within a phonological string.

*   *   *

The test of the correctness of any particular theory of phonological representations will be the predictions it makes about the position of the barycentre given the distribution of attested mass in the phonological space it defines. Theories whose measures of barycentric distances can be correlated with speaker judgements about degrees of affiliation should be valued more highly than those whose measures cannot be so correlated.

2.5 *A Full-scale Phonological Theory*

This section is devoted to developing a full-scale theory of phonological representations, which captures the insight that the phonology is the hash key system used by the language faculty for lexical access, and that phonological representations are curves in a Euclidean space. We assume that this is the only basis for phonological representations, and that they have no special relationship with whatever devices are used to communicate them.

We note firstly that the only difference between our toy phonologies and a more sophisticated one is just that our calculus of phonological representations is simpler than any that would be required for full-scale work.

2.5.1 *Definitions*

In accordance with our radical aims, we introduce the notion of a *Category*. A Category can be viewed as some (phonological) *property* that a position in a string has. There are similarities between Categories and both the elements *and* constituents of Government Phonology, and the motivation for many of the Categories comes from the empirical success of these objects.

Our substantive claims about phonology will turn largely on the definition of the categories actually found in Nature. The choice is essentially axiomatic: nothing in the theory will tell us what categories to look for, or how many there are. The categories used in Natural Language are therefore an arbitrary property of Natural Language as we find it. Relying on the empirical success of the handful of 'categories' used in Government Phonology, incorporating the non-segmentalist proposals of §2.6.2, we stipulate for our theory the following seven Categories:

AXIOM (2.11). CATEGORIES [13]
There are seven Categories, $\kappa = \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}, \mathbf{G}\}$.

There are in Natural Language typically more than just seven possible phonological contrasts in some given position in a phonological representation. Phonological theories since Trubetzkoy 1929 have captured this by allowing the primitive categorial objects to combine to create new objects. In feature theories, features typically come in bundles attached to nodes of various sorts, while in non-feature theories, elements may fuse to form complex expressions. The theory proposed here also allows complex categories. In fact our categorial system is defined simply as the power set of the set of categories {**A**,**B**,**C**,**D**,**E**,**F**,**G**}. In other words, a *phonological category* is any subset of the set of Categories:

Axiom (2.12). Phonological Categories
The phonological categories *are the subsets of* κ.

Recall that the *empty set* is one of the subsets of κ. We dignify the empty set with the special notation '**0**' (read "zero"), using **0** as an alternative notation for $\varnothing$ in phonological diagrams, and when the context of the discussion is Phonological Categories:

Definition (2.13) The Zero Phonological Category
$\mathbf{0} =_{df} \varnothing$.

We are now in a position to define a *phonological string*. We bring together the phonological categories defined above and the notion of 'order encoded in a string' from §2.1.1 and straightforwardly define a phonological string as a string of phonological categories:

Axiom (2.14). Phonological Strings
A phonological string *is a finite sequence of phonological categories.*

In the remainder of this work we shall use the notation **AB**, for example to refer to the phonological category {**A**,**B**}. We shall continue to employ the usual set-theoretic notation for sequences when discussing phonological strings, thus the string ({**A**},{ },{**E**,**F**,**G**}) we shall symbolise (**A**,**0**,**EFG**).

In the next section we define what the minimal phonological string is that can exist independently and be used in the construction of discourse sized utterances. This minimal phonological string will in fact be a *hash key*, as was suggested during the discussion in Chapter One.

2.5.2 *Phonological Keys*

We consider first the *primary hash keys* of Chapter One. The secondary hash keys are discussed in detail in Chapter Three. We exploit the fact that many utterances can be 'broken down' by speakers into minimal 'recognisable subparts', 'words'. Given that these minimal subparts can be elicited individually, we must assume that there are complete phonological strings corresponding to these elicitations. Given also that these individual elicitations allow interlocutors to recover syntactic-semantic information (by recovering an LNode and its associated LObject), we assume that these particular phonological strings are in fact the *primary hash keys* of the Natural Language lookup system, defined in Chapter One. We use this section to characterise just what these minimal phonological strings are, and we shall refer to them as *phonological keys*.

Identifying the set of phonological keys ('minimal words') with the set of hash keys for the lexical look up system has an immediate consequence: the set of phonological keys must be *finite*. Theories which posit 'templates' in the morphology (McCarthy 1979, 1981) impose a similar restriction, although this property is often overlooked. Also significant is the fact that every language seems to 'run out' of 'unanalysable' words. There are languages where this is easier to diagnose than others. Ostensibly templatic languages, as just hinted, but also Chinese-type languages have demonstrably finite repertoires.[14]

We assume, therefore, that phonological keys have a restricted length. In principle, then, there must be two values α and β which correspond to the upper and lower bounds of this length. Lower bounds on 'word length' are frequently encountered in the world's languages, a fact captured in prosodic morphology theories with 'minimal word conditions' (McCarthy & Prince 1990). And a templatic system is a system where all phonological keys have the same length, *i.e.* where $\alpha=\beta$.

Now, assuming that parsing a larger phonological representation (corresponding to some utterance) minimally involves identifying sub-strings which correspond to phonological keys, and hence hash keys that can be 'looked up', we would expect the most efficient choice of upper and lower bounds on phonological keys to be exactly the templatic choice $\alpha=\beta$. The number of candidate parses for each substring would be no greater than α in this case.[15] In the case where α and β differ, much more decision-making has to be made, requiring significantly more computing power, and hence many more candidates (Williams 1994, in prep).

Now, even if these upper and lower bounds, α and β, are *apparently* different, it is still possible to treat the keys whose length they constrain *as if* they were constrained only by a template. That is, we set α to be the same as β, and we 'pad' any keys of length less than β with a null value. This is a common technique in computer architecture, where data is *formatted* so that it complies with a specified template. For example, in a system that manipulates four digit numbers we can include such apparently one-digit objects as 1, 2 and 3 by padding them with leading zeros: 0001, 0002, 0003. We can assume then, that for Natural Languages, the length of phonological keys is fixed. It remains, then, to determine 'how many digits' the Natural Language hash keys have, and how they are padded.

In terms of the phonological categories defined in §2.6.3 we have a natural choice of null value, and that is the empty set, **0** (it is the only phonological category which contains no Categories). So when we build the semantics for phonological keys in terms of integer hash keys (§2.10) we shall ensure that **0** evaluates to 0. In the acoustic interpretation discussed in §2.7 we shall likewise be sure to define the interpretation of **0** as some sort of 'silence'. This is necessary because the role of padding is precisely to make 'shorter' objects apparently 'longer', and if, for example, we wished to claim that both *scrounge* and *it* are phonological keys, we must assume that *it* is 'as long as' *scrounge*. To make 'acoustic sense' of this requires defining the padding as 'silence'.

From the arithmetic properties of integers, however, it is clear that padding zeros cannot just be sprinkled anywhere. It is certainly not the case that 0001 is equivalent to 1000, or that 1010 is equivalent to 1100, or 1001. Given the interpretation function defined in §2.10, which treats a phonological string exactly like a string of digits, we assume that the padding in phonological keys is *leading* (*i.e.* $1 \equiv 0001$). Thus, the phonological key for *it* will have a number of leading **0**-s: **0**…**0***it*. It follows that *it***0**…**0** is *not* an equivalent key to **0**…**0***it*, or *i***0**…**0***t etc.*

It remains to determine the actual key-length used in natural languages. Intuitively this should be the upper bound on the length of an 'unanalysable word'. Recall, however, from Chapter One that this upper bound on key length gives an upper bound on the number of hash keys in the (primary) search space. And this, we know, is intimately connected with the allocation of spatial and temporal computational resources to the lookup system. This in turn has been determined during the evolution of the human brain. From the point of view of the human instantiations of $\mathfrak{H}$-systems, then, the choice of key-length restriction is axiomatic—it depends on no other component of the system.

Given, then, that the key-length is determined by the particular biological allocation of resources to the human lookup system, and given that, to a fair degree of approximation, all human brains are the same, the simplest and strongest assumption we can make is that this natural language key-length is the *same* for all languages. In the following section (§2.7) we develop an interpretive semantics of phonological keys that allows us to extract them from acoustic phenomena, and we shall use this to justify the following assumption, that the key-length in the human lookup system is *four* :

AXIOM (2.15) PHONOLOGICAL KEYS

A phonological key *is a finite sequence of* α *phonological categories. For human language,* $\alpha=4$.

We can begin to motivate this claim by considering, informally, the expressive power of the phonological keys. There are seven Categories, and each position in a phonological key is occupied by a phonological category (a member of the powerset of the set of Categories). Each position in the key can thus be occupied by one of $2^7=128$ phonological categories. Clearly all languages have more than 128 'unanalysable words', so α must be at least 2. When α is 2, there are $128^2=16{,}384$ 'unanalysable words'. This is possibly enough in the case of many languages, but with a string of just 2 phonological categories, it is impossible to define a consistent acoustic semantics. An adequate semantics (for example, one which is able to express an English speaker's intuitions about the phonological properties of the word *scrounge*) only seems to become available when $\alpha=4$. We examine this semantics in detail in the following section.

2.6 *Interpretation*

We tackle the task of relating phonological representations to observables by building an interpretation function that maps them into a simple, yet realistic, characterisation of 'cognitive-phonetic space'.

We further assume that the interpretation function is just that, namely a Tarskian semantics for phonological representations. Such a semantic function is defined for all objects in the language to be interpreted. A theory whose semantics did not meet this criterion would be in the embarrassing position of not being able to relate some of its objects to observables. From a human language point of view, an interpretation function that was unable to process any phonological representation would not be able guarantee that it could process previously unencountered forms. For the human being in the early stages of acquiring a new language, at least, this is patently not the case. It is plausibly not the case for all human beings, since we seem to be able to tackle all sorts of 'strange' and 'foreign' words.

This simple architecture has some nice corollaries. No reference is made to whether or not the phonological representation is a hash key, or if it is a hash key, whether or not it is attested. This means that as speakers of human languages we are able to pronounce and recognise forms which may be novel and/or meaningless. Forms like *blick* are good evidence that this is the case. This also means that insofar as a speaker, **S**, can recognise the models in an utterance generated by some other individual's interpretation function, **S** is able to re-interpret the corresponding phonological categories, delivering the utterance in **S**'s own particular 'voice'. Note further that interpretation functions are specific to *individuals*. Their definition does not rely on any higher hierarchical organisation like 'dialect group' or 'language'. Therefore it does not matter if **S** actually speaks a different language—as long as **S** can recognise models in the utterance, **S** can re-interpret the utterance, not in her own *language*, but in her own native *accent*, without necessarily having any idea what the utterance actually means. Again, one's day-to-day experience would indicate that this is a plausible model: when foreign words are adopted, they are typically subject at least to a change in accent.[16]

We are in broad agreement with the manifesto of linguistic phoneticians, exemplified recently by Ladefoged & Maddieson 1996, Chapter 1, that phonetic space is characterised by a number of 'parameters of variation', along which categorical values are established by individuals. We take a rather different view of how best to create theories of this phenomenon, as discussed in the following section.

2.6.1 *Acoustic models*

The production and perception systems we assume manipulate the acoustic equivalent of Marr's 2½D visual models (Marr 1982), with a limited degree of 'composition', such that for a small, finite set **M** of (acoustic) models, there may be models *m* in **M** which are the 'composition' of two other models *x*,*y* in **M**. We can define the system analogously to the system used to structure the set of phonological categories, namely introduce a set of primitive models, and define an acoustic model to be some subset of this set of primitives. It is these models which we assume do the job of discretising phonetic space.

That this is a plausible characterisation of the structure of linguistically relevant perceptual acoustic space has been demonstrated by several speech recognition studies in 'element based' phonologies (Kaye 1995b, Williams 1992, in prep, Williams & Brockhaus 1992).

Like the models used by other cognitive systems, we assume that our models are defined parametrically, and each individual's repertoire is characterised by given ranges of values for each parameter. Part of the language and group-marking acquisition process obviously involves the acquirer establishing such values.

An example, due to Jackendoff (Jackendoff 1983:85, citing experiments in Labov 1973), of a parametrically defined model is the visual model some of us have of a 'bowl'; there are many parameters, or axes, along which the image of the bowl may be stretched or shrunk, yet it will still be a 'bowl'.

However, if the stretching goes too far along the vertical axis, we suddenly see the image of a 'vase'; if it stretches too far along the horizontal axis, then we may suddenly view it as a 'plate'. As long as parametric values are chosen within the given definitional limits, we will always see a 'bowl', although individual instances of bowls may be characterised by very different parametric values (within the given limits). Similarly our acoustic models can be viewed like the model for the 'bowl'; individual instances may vary quite considerably, but only within the given parametric tolerances.

The actual specification of these acoustic models is a delicate matter. In the best of all possible worlds, which sadly, due to the author's ignorance, this is not, the models would be described by families of differential equations for the energy of the speech signal. This has many benefits, two of which are that it is completely neutral between the mechanics of production and perception, and that it is directly relatable to measurable physical phenomena (making it empirically testable), just as dynamical theories of the weather and the motion of the planets. It also has drawbacks, the chief of which is that it is not conducive to the sort of impressionistic description that is the common currency of most phonological discussion.

Here we shall rely heavily on the informal terminology of tradition to convey the acoustic content of the models, and we shall use a mix of terms appropriate to production and perception, as is common practice. However, we shall always bear in mind that this eclectic and informal approach is simply a narrative device which can and should (with some work) be translated into a mathematically specified dynamical system.

An approach I favour is one where each primitive model is specified by a Fourier series. The combination of primitive models into models (§2.7.2) can be achieved simply by summing the relevant series. The coefficients of the series will provide the locus of the individual specific parametric variation. With the reader's indulgence I shall leave to future research the mathematical details, and ask that the remainder of the chapter be read with the promise of this programme in mind.

2.6.2 *A Theory of Acoustic Interpretation*

In view of the preceding discussion I adopt an 'element-based' approach to the models, assuming that they correspond, in spirit at least, to the realisations of some of the elements and constituents of Government Phonology (Kaye, Lowenstamm & Vergnaud 1990, Harris & Lindsay 1995). It is important, however, that the distinction be made between objects in our phonology (phonological categories and keys) and these models, *which are not part of the phonology*. They are simply one set of objects used to realise, or interpret the phonological objects. The architecture argued for in Chapter One makes it possible that there are other cognitive systems which can interpret phonological objects – sign language and written language being two such examples.[17]

The interpretation function we assume is a semantics for phonological representations, as already stated. Accordingly we need to introduce the semantic analogues of our phonological representations:

Axiom (2.16). Primitive Models
There is a set of primitive models, $\mu = \{R,A,H,D,P,T,J\}$.

Axiom (2.17) Acoustic Models
The acoustic models *are the subsets of* μ.

We see no compelling reasons to depart from the claims of theories like Government Phonology that in acoustic terms, human beings use a three-element 'resonance' space (A,I,U), a two-element 'tonal' space (L,H) and a two-element 'airstream/laryngeal' space (?,h) when interpreting phonological expressions (Kaye, Lowenstamm & Vergnaud 1990, Harris & Lindsay 1995). It is, in any case, uncontroversial that the human vocal tract is able to generate a variety of 'source' airstreams, to which additional 'filters' are applied (Ladefoged 1962).

However, to underline the difference in kind between our models and the elements of Government Phonology, we use a different set of acoustic criteria, while keeping to the same 'physiological geometry' of three models for resonance filters, and four for sources.

To illustrate the approach, we concentrate here on one particular dimension, or axis, of phonetic space, and show how we imagine the models discretise that axis. We assume that one of the axes of phonetic space is a continuum which varies from silence at one end (no energy), to 'vowels' at the other (high, periodic energy). This dimension has much in common with a category traditionally called 'Manner', and various impressionistic labels have been given to its subparts. Ladefoged & Maddieson (*op.cit.*) describe the categories 'stop', 'fricative', 'approximant', 'vowel' and 'tap/trill' as existing on the Manner axis. In our more mathematically orientated approach we prefer to envisage this dimension as one which is characterised by 'amount of periodic energy'.

Now, with a single model we can only discretise a given continuum into two categories. This is plainly inadequate for the Manner dimension, as we believe that all languages make use of more than just stops and vowels from this dimension. Using two models allows us to discretise the continuum into four. We assume that this is adequate, and that using three models (giving an eight-way categorisation of Manner) is not a realistic characterisation of individually employed Manner categories.

So we select two primitive models and ensure that their (mathematical) definitions conspire to produce the acoustic description of the Manner dimension. We accordingly define A as 'high, periodic energy', and R as 'low, periodic energy'. Their combination naturally falls out as periodic energy somewhere between high and low. The absence of both models is naturally a description of silence. By superimposing the articulatory categories of Ladefoged & Maddieson, we can see how models involving A and R are likely to be realised.

Definition (2.18). Primitive Models A and R

(a) stop   tap   fricative   approximant   tense vowel   lax vowel
(b) ————————————————————————————
(c) 0 ———   — R ————————   —RA ————   —— A
(d) —   —   ——   ——

In the table above we show (*a*) Ladefoged & Maddieson's labels; (*b*) the acoustic continuum, going from no energy at the far left, to high, periodic energy on the far right; (*c*) a schematic representation of the (universal) limits of the extent of the models A and R (this is analogous to the 'bowl' example. The black lines indicate where you can be in the continuum and still be a possible 'bowl'); (*d*) a schematic representation of how an individual interpretation function might instantiate the models (the limits of an individual's accent).[18]

The definition of the other acoustic models proceeds analogously, by identifying a plausible dimension, and then selecting enough models to discretise that dimension into a number of categories that it is supposed can be maximally supported by a single individual. The inadequacies of existing phonetic terminology make the following discussion more intricate than the underlying reality (and, it is hoped, the eventual mathematical specification of the models). This is because several apparently well-motivated phonetic dimensions are defined only with respect to specific other dimensions, rather than completely generally.

Thus there is a dimension of 'Place' which specifies where non-vowels are articulated; yet there is a completely separate dimension which specifies resonance properties, but it is only applied in the description of vowels. In such cases, we have endeavoured to follow the lead of theories like Government Phonology which attempt to unify these separate dimensions into universally applicable ones, for example by equating the Place dimension in consonants and the resonance dimension in vowels. One unfortunate upshot of this is that it makes the definitions to be given below look more ‘conditional’ than they really are.

Definition (2.19). Primitive Models T and P

	0 ——	—— T ————	—— TP ——		—— P
Place	sub-apical	apical	retroflex	laminal	bilabial
Vowels	a/ɑ	ɨ/ʌ	o/ɔ		u/ʊ

Together, the (T,P)- and (A,R)-systems define a whole class of attested phonetic phenomena.

The 0-model, the interpretation of **0**, we assume is an acoustic ‘identity’. Given that we need a fixed key-length, and given that acoustic objects which demonstrably interpret keys seem to vary in length (*cf. sprint* and *it*) this is an essential assumption. There is no particular IPA symbol for this, but we shall assume that the acoustic function is simply to ‘do nothing extra’, which in many cases falls out to be ‘just continue with processing what you are already processing’. We are saying, in other words, that the human language processing system is blind and deaf to the 0-model. We let our phonetic notation mimic the acoustic phenomena by not symbolising the 0-model at all, leaving the location of **0**-categories to the phonological parser (*q.v.* §2.7).

Let us summarise the combinations of the primary models. We choose to use letters from the International Phonetic Alphabet to illustrate possible realisations of these models, but the reader should always bear in mind that an individual interpretation function is defined by selecting some range of values within the universally given parametric limits (as illustrated in Definition 2.18 above). In individual cases this may result in an acoustic phenomenon which a trained phonetician might well symbolise differently. Accordingly, the values in the table below and those following should be read as nothing more than a particular individual instantiation of a phonetic interpretation function which has been chosen to give a sort of ‘average’ feel for what sort of acoustic phenomena are hypothesised to attend the given models.

Example (2.20).

	0	T	TP	P
0		t	ʧ	p
R	r	l	ɫ	w
AR	ɑ	ɨ	o	u
A	ʌ	a	ɔ	ʊ

We move on to J. The dimension we identify with J is that associated with ‘high’ and ‘front’ in vowels, and ‘palatal’ and ‘palatalised’ in non-vowels.

Definition (2.21). Primitive Model J

0 ———————— ———————— J

Vowels	low	retracted/lowered	advanced/raised	high/front
Non-vowels	un-palatalised	retracted	advanced	palatalised

Note that from our definitions the model J is some sort of sub-apical stop (0) that has been ‘advanced’ by J, which we here plausibly identify with k.

Example (2.22). Primary Models with Secondary Model J

	J	TJ	TJP	JP
0	k	t^j	c	p^j
R	j	ʎ	ɭ	ɥ
AR	i	e	ø	y
A	ɪ	ɛ	œ	ʏ

Finally we turn to the models D and H. We assume that these two models discretise the continuum of what may be loosely called ‘aperiodic energy’ or perhaps ‘high frequency energy’. At one extreme we have no aperiodic energy, which in vowels we may assume simply means ‘modal voice’, in stops ‘un-aspirated’, and in fricatives ‘voiced’. As we go up the scale, we imagine aperiodic turbulence being added. This may manifest itself as increased ‘damping’ of already periodic energy (thus ‘nasals’ can be seen as a damped form of fricative; nasalisation of vowels can be similarly viewed), leading to increased disruption of periodic energy (‘voicelessness’, ‘sibilancy’, ‘aspiration’), and/or amplification of high frequency energy. Possibly controversially, following some phonologists, we choose to locate 'tone' (and other vocalic 'modalities' such as 'creak' and 'breathy voice') in this continuum, equating 'low tones' with damped periodicity, and 'high tones' with higher frequency energy.

DEFINITION (2.23). PRIMITIVE MODELS D AND H

0 —— —— D ——————— ——DH—— —— H

Vowels	modal	nasalised	low-tone	creak	high-tone	devoiced
Fricatives	voiced	nasal	liquid	voiceless		sibilant
Stops	unvoiced	implosive	voiced		spirantised	aspirated

EXAMPLE (2.24). A FULLY SPECIFIED INTERPRETATION FUNCTION

	0	T	TP	P	J	JT	JTP	JP
0		t	ʧ	p	k	t^{j}	c	p^{j}
D	r̃	n	ɴ	m	ŋ	ɲ	ɳ	ɱ
DH	ʔ	ð	ʒ	β	ɣ	z	ʝ	v
H	ħ	t^{h}	ʧ^{h}	p^{h}	k^{h}	t^{jh}	c^{h}	p^{jh}
R	r	l	ɬ	w	j	ʎ	ɭ	ɥ
RD	z	d	ʤ	b	g	d^{j}	ɟ	b^{j}
RDH	s	st	sʧ	sp	sk	st^{j}	sc	sp^{j}
RH	χ	θ	ʃ	ɸ	x	z̥	ç	f
A	a	ʌ	ɔ	ʊ	ɪ	ɛ	ɶ	ʏ
AD	ã	ʌ̃	ɔ̃	ʊ̃	ɪ̃	ɛ̃	ɶ̃	ʏ̃
ADH	á^{n}	ʌ́^{n}	ɔ́^{n}	ʊ́^{n}	ɪ́^{n}	ɛ́^{n}	ɶ́^{n}	ʏ́^{n}
AH	á	ʌ́	ɔ́	ʊ́	ɪ́	ɛ́	ɶ́	ʏ́
AR	ɑ	ɨ	o	u	i	e	ø	y
ARD	ɑ̃	ɨ̃	õ	ũ	ĩ	ẽ	ø̃	ỹ
ARDH	ɑ́^{n}	ɨ́^{n}	ó^{n}	ú^{n}	í^{n}	é^{n}	ǿ^{n}	ý^{n}
ARH	ɑ́	ɨ́	ó	ú	í	é	ǿ	ý

Finally, I also assume, uncontroversially, that a model is realised in 'real' time (i.e. 'short' and finite), and that models follow each other in a flow of time. Let us call a string of such models so realised a *stream*:

AXIOM (2.25). MODEL STREAMS

A (model) stream *is a finite sequence of acoustic models.*

In building our interpretive model for phonological representations, then, we are constructing a map from the set of phonological representations into the set of streams. I shall use the symbol [ ] for this mapping, writing Σ=[S] to mean the model stream Σ is the interpretation of phonological representation S:

AXIOM (2.26). SEMANTICS FOR PHONOLOGICAL REPRESENTATIONS

The phonetic interpretation *of a phonological representation* S, *written* [S], *is given by the function* [ ]:S*→Σ*, *where* S* *is the set of all phonological representations, and* Σ* *is the set of all streams of models.*

In particular, for a phonological representation $S=(\kappa_1,\ldots\kappa_n)$, $[S]=([\kappa_1],\ldots[\kappa_n])$, *where for any phonological category* κ, *for all categories c in* κ, [κ] *is the minimal set satisfying* $(c\in\kappa \Leftrightarrow I(c)\in[\kappa])$, *where I is a one-to-one function* $I{:}\kappa\rightarrow\mu$ *from the categories to the models such that* $I(\mathbf{A})=_{df}R$, $I(\mathbf{B})=_{df}A$, $I(\mathbf{C})=_{df}H$, $I(\mathbf{D})=_{df}D$, $I(\mathbf{E})=_{df}T$, $I(\mathbf{F})=_{df}P$, $I(\mathbf{G})=_{df}J$.

We can illustrate this function quite straightforwardly. Take the phonological representation (**AB**,**CD**,**ADE**,**G**), a form not attested for the author, and some distance from his barycentre; its interpretation according to the above definition is simply (RA,HD,RDT,J), which, using the hypothetical realisations of the above discussion, might be realised ɑʔnk.

2.6.3 *Acoustic Fine-structure*

Before we leave this description of phonetic interpretation, we pursue here one possible avenue which allows us to capture the methodological *desideratum* mentioned earlier, that the sort of segments identified by phoneticians are not exactly the same as phonological 'segments', and that phonological segments are sometimes realised by acoustic objects larger than a phonetician's segment (§2.6.2). We capture this in our semantics by providing a phonetic 'fine-structure'. We assume that the time over which a phonological category is interpreted defines a phonetic *packet*. In the structure of this packet we identify three consecutive *phases: attack, sustain,* and *release* phases, which we shall call the A-, S- and R-phases. A primitive model is realised in a time frame that may occupy one or more (consecutive) phases. For any primitive model, then, there are available a potential six phasing patterns: realised in A, in S, in R, through A and S, through S and R, and through A, S and R. We further assume (as our terminology suggests) that the A-, S- and R-phases of the packet define an *envelope* which constrains the energy functions of models realised therein. Crudely, the A-phase defines a positive energy gradient, the S-phase defines a constant energy gradient, and the R-phase defines a negative energy gradient.

By definition, the S-phases are the most salient in the signal, being the periods of greatest stability. We suspect that it is the S-phases which give naïve speakers whatever intuitions they may have about the segmentising of the speech signal. Phonetic training seems to make us more aware of the A- and R-phases. As field linguists we become tempted to segmentise in greater detail, giving separate symbols to A- and R-phase realisations as well as S-phase ones.

Now, speech is the interpretation of a sequence of phonological categories, and we assume that the result is a sequence of phonetic packets. However, we assume that the packets are not realised strictly independently of each other. The reasons for this we do not know, but assume that it is an imperfection in the adaptation of the oral tract for speech. We assume that only the S-phases must always be realised distinctly, and that in a sequence of packets, the R-phase of one packet 'overlaps' the S-phase of the following packet. The energy function of this overlapping we assume is simply the sum of the energy functions of the models in this overlap-phase, and that the envelope is simply the sum of the R- and A-envelopes. Given what we have said, that the R-phase is a decreasing gradient, and that the A-phase is an increasing envelope, we expect the envelope of the overlap-phase to be similar to the S-phase (summing a negative gradient with a positive gradient tends to lessen the total gradient).

It is this similarity in the envelope which points to the imperfect adaptation. If our perception mechanisms are indeed attuned to detecting the S-phases in a speech signal, and if the overlap-phase can approach an S-phase-like envelope, then there is the danger that when there is enough energy in the overlap-phase, it might get confused with an S-phase. We discuss the implications of this further in §2.7.

However, we assume that in general the perception and production mechanisms are sensitive enough to be able to distinguish even S-phase from (R+A)-phase realisations. Possible confusion arises, we suppose, either from a pathology in the signal, or through lack of sophistication on the part of the investigator. It is this latter state that we find ourselves in most often, and it is this latter state which 'broad' (and other so-called 'systematic') transcriptions and orthographies incline us towards. It thus becomes hard sometimes to convince oneself to rely on actual acoustic facts.

We suppose that this level of detail and approximation is sufficient for the purposes of this work. In many respects it goes far beyond the detail usually considered sufficient for phonological analyses, and provides a more direct mapping to the fine-structure acoustic signal. Thus a single phonological category (crudely, a traditional phonologist's 'segment') is detectably realised, according to our proposals, by a structure with up to three phonetic events of varying extent and intensity.

Example (2.26). The Fine-structure of Acoustic Packets

Envelope	A		S		R
Narrowest transcription		$\mathbf{s}_1$	$\mathbf{s}_2$	$\mathbf{s}_3$	
Narrower transcription			$\mathbf{s}_1$		

In the rather clumsy terminology of tradition, we might say that in the mechanism of overlapping R- and A-phases we have a locus for automatic contact, or 'assimilation', phenomena. But it is most important to bear in mind that these micro-level phonetic phenomena are properties, side-effects, only of the phonetic interpretation function. They make no reference to the phonology, and, we may suppose, are directly caused by the physical mechanisms used to instantiate the interpretation function. An important diagnostic in this case is that these phenomena should be linguistically irrelevant, and except to trained phoneticians, should be sub-conscious and pass largely un-noticed. And they should be completely general, and exceptionless. There is no sense in which they represent (phonological) categorical changes, since that would imply that they were 'conscious'. Our phonological categories are defined, after all, to provide speakers with a means of differentiating meanings; changing a phonological category must therefore alert the speaker/hearer to a possible change in meaning, hence cannot be considered a sub-conscious process, by any stretch of the imagination.

This device contains a hidden assumption, and that is that two acoustic phenomena which are differentiated only by model-phasing can never be used by a single individual to interpret two distinct phonological categories. The individual must 'choose', as it were, one or other realisation (*q.v.* §2.7). Different choices we can roughly equate with differences in 'accent'.

In this way, certain assumptions about what are believed to be impossible contrasts in the world's 'phonologies' can be seen simply as a side-effect of the interpretation function. The particular choices we have made here for our interpretation function ultimately may not be successful; that is by the way. The point that remains is that such devices seem to be an obvious locus for stating certain generalisations about possible linguistically relevant *acoustic* contrasts, without needing to stipulate them in the *modality-neutral* phonology.

2.6.4 *Example—Nasal Place Assimilation*

Take as an example a pervasive phenomenon, which we shall illustrate from English, called 'nasal place assimilation' (NPA). The phenomenon involves the apparent change of the place of articulation of preconsonantal nasals to the point of articulation of the following consonant. If we assume that in English the D model is realised in the R-phase of all vowel models,[19] then the mechanics of the speech signal, as defined above, mean that any A-phase models in the immediately following model will overlap this D. In the case of a following P, for example, the point of contact will be realised with the combination DP, some kind of m.

NPA, then, rather than posing a problem for a rule-free phonology, can actually be seen as an important illustration of the structure of the acoustic interpretation function. Notice that the existence of NPA does not in itself provide a deciding case for whether NPA is a by-product of a phonological process or a phonetic (better, interpretive) one. However, claiming that it is an epiphenomenon in the interpretive module makes certain predictions: that speakers/hearers will not be aware of any (phonological) categorial change (true: untrained English speakers are quite convinced that they say/hear tɛn in tɛmpɪn *tenpin*);[20] that the acoustic properties of the assimilated nasal are not necessarily the same as those of the corresponding 'real' nasal, that is, one realised largely in the S-phase of packet (true: the energy profile of the *m* in *camp* is different from that of the *m* in *mother*. Most obviously it is consistently less energetic); and that the process should be completely exceptionless. Claiming that NPA is the result of a phonological process (*i.e.* involves changing phonological categories) would on the face of it predict precisely the opposite in these cases.

2.7 *Signal Parsing*

The ASR-envelope theory implies that there may be several candidate parses of an model stream. We examine the theoretical entailments briefly here, and illustrate the analytical process in great detail in §2.8.

2.7.1 *Extracting Models*

The model given above provides a great deal of acoustic detail, and is based on an assumption that we perceive the acoustic detail in discrete *slices*. However, our perception, both as individual language acquirers and certainly as investigative scientists, seems to be both keenly sensitive and wilfully purblind. With training we can easily convince ourselves that we perceive all the micro-structure implied by the ASR-model; however, we also spend much of the time apparently ignoring this detail. In particular, as speech rate increases certain aspects of the signal, namely the transitions and non-S portions of the envelope tend to get temporally 'squashed'. On the other hand, when speech is made slow and deliberate, the non-S portions of the envelope tend to get over emphasised to the extent that the envelope gradients become levelled.

This state of affairs we must assume is a result of an imperfect match between the perception and production mechanisms. Consequently we as acquirers, users and investigators must be aware that several candidate parses of the acoustic signal may be available. This is not a controversial assumption, and can be readily exemplified from LE. Is *glimpse* glɪms or glɪmps? The answer suggested by our model is that it is both (and possibly several others). Each candidate is generated by emphasising, or suppressing, certain non-S portions of the ASR-envelopes.

The pegs onto which we can hang these candidates are the prototypic realisations of an individual speaker's models. These prototypes constrain the appearance of model-combinations throughout a packet. A given acoustic analysis may in fact contradict the speaker's prototypes, in which case it can be discounted. Any candidate which survives this consistency checking can be declared a successful parse.

We note that there may well be some acoustic phenomena for which there is more than one successful parse. In this case we need an analytical tool to help us decide. We explore this in detail in §2.8 below (*cf. also* §2.7.2).

We are careful to recall the definition of these contact phenomena. The device is not a back-door for the introduction of arbitrary transformational devices. In fact, careful reading of the definition should convince the reader that no transformation is actually sanctioned at all, merely the 'epenthesis' of an object with properties derived from properties of its immediate surroundings. In fact, introducing transformational mechanisms would destroy the isomorphism that is necessary if we wish the phonetics to be a Tarskian semantics for the phonology, so we have been particularly careful to restrict the formal power of our model theory. It is gratifying to see that even within the extremely simple limits that this imposes, a great variety of acoustic phenomena can be described, the basis of a theory of individual variation has been laid, and a handful of pervasive automatic contact phenomena can be expected.

2.7.2 *Recovering Phonological Categories*

A typical analytical problem derives directly from our definition of 'nothing' (the 0-model) as the interpretation of the phonological category **0**.

Given an acoustic event, which interprets a phonological key, in which we can identify a single phonological category *k*, we have the following candidate analyses: **000***k*, **00***k***0**, **0***k***00**, *k***000**. In the case of two phonological categories *j* and *k* we have as candidates: **00***jk*, **0***jk***0**, *jk***00**, **0***j***0***k*, *j***00***k*, *j***0***k***0**.

The final choice of analysis can only be made, according to our postulates and by the resulting methodology, by calculating the barycentric shifts entailed by adopting each of the candidates in turn, and selecting the form which entails the least barycentric shift. That is, there is no universally correct analysis of the phonetic event, and there is nothing in the signal which could possibly provide a criterion for selection.

In §2.8 we pursue an illustrative example in some detail. Before leaving the current section, however, we look briefly at one of the other tasks of analysis/acquisition, and that is the establishment of an internally consistent interpretation function.

2.7.3 *Semantic Consistency*

A common analytical problem can arise with a phonetic event in which a sequence of two models *mn* is identified. Within the definition of the acoustic interpretation function, it may be possible to analyse this sequence as a sequence of two corresponding phonological categories *jk*, or it may be that the sequence *mn* is a phase-sequence of the ASR-envelope of a single model ($m+n$), and hence is the interpretation of a single phonological category. In the former case we then have to consider the following phonological analyses: **00***jk*, **0***jk***0**, *jk***00**, **0***j***0***k*, *j***00***k*, *j***0***k***0**, as discussed above.

The latter case arises when the analyst (or acquisition device) has imperfect knowledge of a particular interpretation function, and so we assume that the analyst has available a method to check the semantic consistency of the ASR-analyses.

For example, imagine, that we encounter a form glɪntʰ, which we feel certain (either from external evidence, or simply by hypothesis) needs to be analysed as a single phonological key, and assume further that we are confident in the analysis of the g, l, ɪ and tʰ occurring at t=1…4 respectively.

Let $\mathbf{g}_1$ be the phonological category realised as g, and define $\mathbf{l}_1$, $\mathbf{i}_1$ and $\mathbf{t}_1$ *mutatis mutandis*. The model content of n (DT) remains to be distributed, somehow, between packet 3 and packet 4. Whatever models are assigned to packet 3 must then be the interpretation of corresponding Categories which belong to the phonological category at $t=3$. Thus we have that at $t=3$, the phonological category is given by $\mathbf{i}_1+\boldsymbol{X}$; similarly, the actual phonological category at $t=4$ is given by $\mathbf{t}_1+\boldsymbol{Y}$, where the interpretation of $\boldsymbol{X}+\boldsymbol{Y}$ is DT ($[\boldsymbol{X}+\boldsymbol{Y}]=$DT). All possible solutions are:

#	*X*	*Y*	#	*X*	*Y*	#	*X*	*Y*
(a)	**0**	**DE**	(d)	**E**	**D**	(g)	**E**	**DE**
(b)	**D**	**E**	(e)	**DE**	**0**	(h)	**DE**	**E**
(c)	**D**	**DE**	(f)	**DE**	**D**	(i)	**DE**	**DE**

Now, any solution where **E** is in ***X*** entails that there should be no sequence of two phonological categories at *t*=3,4 in a single phonological key whose interpretation is ɪmpʰ, since that sequence must, by definition, give ɪʀpʰ, where ʀ is *exactly* the same as the ʀ in ɪʀʧʰ. The analyst will quickly discover plenty of counter-examples (such as lɪmpʰ and lɪʀʧʰ), so we can say that solutions (d–i) do not lead to a consistent acoustic semantics, given the facts of LE.

Note that this method does not require subtle analyses, or intricate logical deductions and forecasts. What it does is to ensure that as more and more novel forms are encountered, they are assigned interpretations which are consistent with the interpretations of other forms. On the whole this seems to require nothing more than encountering a deciding case in the environment. If a deciding case is not to be found, we can assume that the choice of analysis is made according to the barycentric divergence of the candidates (among semantically consistent candidates, the least weird is preferred).

In our example, there is only one consistent analysis. Solution (b) ('postnasalised vowels') entails that LE could never use 'nasalised vowels', since the same phonological category is responsible for both. Solutions (a) and (c), on the other hand entail that the phonological category **DE** be realised ⁿd, which means that there would be no way to analyse ɪd, since it would have to be realised ɪnd. There are may counter-examples for the LAD/analyst to find, such as lɪd, so we must reject (a) and (c) in favour of (b): 'postnasalised vowels'.

Some anecdotal evidence that (b) is indeed the correct analysis for LE is that it entails that when a model stream that contains 'nasal vowels' is being processed, our LE interpretation function would 'Anglicise' them to 'postnasalised vowels'. This seems to be true (a French form like *bon mot* bɔ̃mo becomes bɔmmʌw, where both forms have *exactly* the same phonological representation, as recovered from the model-content of the stream).

Another perhaps more speculative piece of evidence comes from the history of LE. At the time of the Norman invasion of 1066AD, the local English speaking community was exposed to a huge quantity of Norman French. A vast portion of present-day LE can be traced back to this exposure (Blake 1992). That is, the linguistic contact was large and prolonged enough to create a significant barycentric shift to some middle-ground between Anglo-Saxon and Norman French. It is surprising, then, that although the vowel system in particular is claimed to have been the greatest victim of this barycentric shift, there are no nasal vowels in LE, Norman French almost certainly having them (Pope 1934:§§1149–52).

It is a surprise, that is, only if we adopt solutions (a) or (c): 'prenasalised consonants'. In this case the phonological categories which can be interpreted as nasal vowels (and which we would have to suppose were languishing unattested in Anglo-Saxon far from the barycentre) would be available for attesting Norman French forms with nasal vowels.

If, however, we adopt solution (b): 'postnasalised vowels' we have an immediate answer. Because the phonological categories responsible for nasal vowels are precisely the same as those responsible for 'postnasalised vowels', the local English would have had no choice but to pronounce the nasal vowels as 'postnasalised vowels'. The same is still true today.

* * *

Having now defined a rudimentary interpretation function that gives us some handle on the relationship between phonological representations and observable acoustic phenomena, we are in a position to consider the phonological keys typically used by speakers of human languages. We discuss this in the next section (§2.8) with reference to London English (the author's native language).[21] This serves as a vehicle for illustrating certain fundamental properties of the phonology, including additional support for the claim that the key length in the human language lookup systems is four.

2.8 *London English*

In order for language acquisition to take place, we must assume that the acoustic interpretation function is already sufficiently mature (or sufficiently specified in the genotype) to be able to recover acoustic models from the acoustic environment. If this were not the case, there would be no way for the L[anguage] A[cquisition] D[evice] to bootstrap itself with linguistic stimuli. As linguists attempting to unravel the phonological representations of some language, we are in the same position as the LAD. From the model presented in §2.7 we are now in the position of having a 'mature' acoustic interpretation function, which should enable us to make a good approximation of the likely model content of the acoustic environment, and hence a good approximation of the communicated phonological categories. Our analysis is consequently constrained by the same postulates as those we believe constrain the LAD. For our purposes here this means that from among competing phonological analyses, the analysis which is chosen is the one which is least divergent from the current barycentric function.

It is important that we are aware of the sources of possible ambiguities. They derive from both genuine phonological ambiguities, and from ambiguities generated by the interpretation functions (§2.7). Phonological ambiguities are minimal. They consist exactly of the number of ways in which it is possible to analyse a phonological string into 'stems' and 'affixes'. In the ensuing discussion we shall assume that this problem has been resolved by considering as far as is possible, without anticipating too much the analysis of Chapter Three, forms with no apparent affixes.

Interpretive ambiguities result, as discussed in the preceding section, from the imperfect match between the perception and production mechanisms in human speech. The result is that a single acoustic event may give rise to a number of possible phonological analyses. The candidates are ranked according to their barycentric similarity. The analysis chosen is the barycentrically least divergent curve.

We assume also that the method to be outlined here presupposes the existence of a consistent interpretation function, the establishment of which is itself an important part of the overall process of acquisition/analysis (§2.7).

2.8.1 *Estimating the Barycentric Function*

We begin our analysis as the LAD might begin acquiring LE. Imagine one of the first forms we encounter is ɪtʰ *it*. From the definition of the acoustic interpretation function we can safely assume that the model-content of this event is AJ followed by TH. The corresponding phonological categories are **BG** and **CE**. Let us symbolise these phonological categories $\mathbf{i}_1$ and $\mathbf{t}_1$ respectively. The candidate phonological parses for ɪtʰ are then $\mathbf{00i_1t_1}$, $\mathbf{0i_1t_10}$, $\mathbf{i_1t_100}$, $\mathbf{0i_10t_1}$, $\mathbf{i_100t_1}$, $\mathbf{i_10t_10}$. Since we have no other attested forms, we don't have any way of ranking these candidates, so we keep them all 'alive' as possible starting points. Since we have only one attested form, each of these hypothetical points represents additionally a possible starting point for the barycentre.

Now imagine that we encounter another form bɪtʰ *bit*. The acoustic analysis provides us with the models (DP,AJ,TH), with corresponding phonological categories **DF**, $\mathbf{i}_1$ and $\mathbf{t}_1$ respectively. Let us symbolise **DF** with $\mathbf{b}_1$. Our candidates for phonological analysis are $\mathbf{0b_1i_1t_1}$, $\mathbf{b_10i_1t_1}$, $\mathbf{b_1i_10t_1}$, $\mathbf{b_1i_1t_10}$. Now we can calculate the barycentric divergences entailed by adopting each of these candidates with respect to each of the hypothetical barycentres given by the *it*-analysis. We shall choose the analyses of *it* and *bit* which minimise the barycentric shifts.

For example, take the candidates $\mathbf{00i_1t_1}$ and $\mathbf{0b_1i_1t_1}$, where the barycentre is at $\mathbf{00i_1t_1}$ $(\mathbf{0},\mathbf{0},\mathbf{BG},\mathbf{CE})$. We first calculate all the differences in the barycentric curve and all the differences in the $\mathbf{0b_1i_1t_1}$-curve:

Example (2.27a). Differences.

$\boldsymbol{f}=(\mathbf{0},\mathbf{0},\mathbf{BG},\mathbf{CE})$ $[\mathbf{00i_1t_1}]$

i	$\delta_i^0 \boldsymbol{f}$	$\delta_i^1 \boldsymbol{f}$	$\delta_i^2 \boldsymbol{f}$	$\delta_i^3 \boldsymbol{f}$
1	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{BG}$	$\mathbf{CE}-3\mathbf{BG}$
2	$\mathbf{0}$	$\mathbf{BG}$	$\mathbf{CE}-2\mathbf{BG}$	
3	$\mathbf{BG}$	$\mathbf{CE}-\mathbf{BG}$		
4	$\mathbf{CE}$			

$\boldsymbol{g}=(\mathbf{0},\mathbf{DF},\mathbf{BG},\mathbf{CE})$ $[\mathbf{0b_1i_1t_1}]$

i	$\delta_i^0 \boldsymbol{g}$	$\delta_i^1 \boldsymbol{g}$	$\delta_i^2 \boldsymbol{g}$	$\delta_i^3 \boldsymbol{g}$
1	$\mathbf{0}$	$\mathbf{DF}$	$\mathbf{BG}-2\mathbf{DF}$	$\mathbf{CE}-3\mathbf{BG}+3\mathbf{DF}$
2	$\mathbf{DF}$	$\mathbf{BG}-\mathbf{DF}$	$\mathbf{CE}-2\mathbf{BG}+\mathbf{DF}$	
3	$\mathbf{BG}$	$\mathbf{CE}-\mathbf{BG}$		
4	$\mathbf{CE}$			

We next calculate the differences between corresponding cells in the above tables, and finally sum the squares of their magnitudes to arrive at the measure of divergence Δ between $\mathbf{0b_1i_1t_1}$ and the candidate barycentre $\mathbf{00i_1t_1}$.

Example (2.27b). Measure of divergence

Differences

i	$\delta_i^0 \boldsymbol{g}-\delta_i^0 \boldsymbol{f}$	$\delta_i^1 \boldsymbol{g}-\delta_i^1 \boldsymbol{f}$	$\delta_i^2 \boldsymbol{g}-\delta_i^2 \boldsymbol{f}$	$\delta_i^3 \boldsymbol{g}-\delta_i^3 \boldsymbol{f}$
1	$\mathbf{0}$	$\mathbf{DF}$	$-2\mathbf{DF}$	$3\mathbf{DF}$
2	$\mathbf{DF}$	$-\mathbf{DF}$	$\mathbf{DF}$	
3	$\mathbf{0}$	$\mathbf{0}$		
4	$\mathbf{0}$			

Squares of magnitudes of differences

i	$\lvert\delta_i^0 \boldsymbol{g}-\delta_i^0 \boldsymbol{f}\rvert^2$	$\lvert\delta_i^1 \boldsymbol{g}-\delta_i^1 \boldsymbol{f}\rvert^2$	$\lvert\delta_i^2 \boldsymbol{g}-\delta_i^2 \boldsymbol{f}\rvert^2$	$\lvert\delta_i^3 \boldsymbol{g}-\delta_i^3 \boldsymbol{f}\rvert^2$
1	0	2	8	18
2	2	2	2	
3	0	0		
4	0			

Measure of divergence

$\Delta(\boldsymbol{g}-\boldsymbol{f})=\sqrt{(0+2+0+0+2+2+0+8+2+18)}=\sqrt{34}\approx 5.83$

We perform this calculation for each candidate barycentre, and each candidate analysis of bit^h. The results are summarised in the following tables.

Example (2.28). Candidate Analyses

$\mathbf{0b_1i_1t_1}$ (**0**,**DF**,**BG**,**CE**)

Candidate Barycentre		*Measure of divergence*	
$\mathbf{00i_1t_1}$	(**0**,**0**,**BG**,**CE**)	√34	≈5.831
$\mathbf{0i_10t_1}$	(**0**,**BG**,**0**,**CE**)	√156	≈12.49
$\mathbf{i_100t_1}$	(**BG**,**0**,**0**,**CE**)	√60	≈7.746
$\mathbf{0i_1t_10}$	(**0**,**BG**,**CE**,**0**)	√224	≈14.97
$\mathbf{i_1t_100}$	(**BG**,**CE**,**0**,**0**)	√86	≈9.274
$\mathbf{i_10t_10}$	(**BG**,**0**,**CE**,**0**)	√126	≈11.22

$\mathbf{b_10i_1t_1}$ (**DF**,**0**,**BG**,**CE**)

Candidate Barycentre		*Measure of divergence*	
$\mathbf{00i_1t_1}$	(**0**,**0**,**BG**,**CE**)	√8	≈2.828
$\mathbf{0i_10t_1}$	(**0**,**BG**,**0**,**CE**)	√132	≈11.49
$\mathbf{i_100t_1}$	(**BG**,**0**,**0**,**CE**)	√34	≈5.831
$\mathbf{0i_1t_10}$	(**0**,**BG**,**CE**,**0**)	√198	≈14.07
$\mathbf{i_1t_100}$	(**BG**,**CE**,**0**,**0**)	√92	≈9.592
$\mathbf{i_10t_10}$	(**BG**,**0**,**CE**,**0**)	√100	=10

$\mathbf{b_1i_10t_1}$ (**DF**,**BG**,**0**,**CE**)

Candidate Barycentre		*Measure of divergence*	
$\mathbf{00i_1t_1}$	(**0**,**0**,**BG**,**CE**)	√116	≈10.77
$\mathbf{0i_10t_1}$	(**0**,**BG**,**0**,**CE**)	√8	≈2.828
$\mathbf{i_100t_1}$	(**BG**,**0**,**0**,**CE**)	√78	≈8.832
$\mathbf{0i_1t_10}$	(**0**,**BG**,**CE**,**0**)	√78	≈8.832
$\mathbf{i_1t_100}$	(**BG**,**CE**,**0**,**0**)	√94	≈9.695
$\mathbf{i_10t_10}$	(**BG**,**0**,**CE**,**0**)	√142	≈11.92

$\mathbf{b_1i_1t_10}$ (**DF**,**BG**,**CE**,**0**)

Candidate Barycentre		*Measure of divergence*	
$\mathbf{00i_1t_1}$	(**0**,**0**,**BG**,**CE**)	√198	≈14.07
$\mathbf{0i_10t_1}$	(**0**,**BG**,**0**,**CE**)	√78	≈8.832
$\mathbf{i_100t_1}$	(**BG**,**0**,**0**,**CE**)	√142	≈11.92
$\mathbf{0i_1t_10}$	(**0**,**BG**,**CE**,**0**)	√8	≈2.828
$\mathbf{i_1t_100}$	(**BG**,**CE**,**0**,**0**)	√184	≈13.56
$\mathbf{i_10t_10}$	(**BG**,**0**,**CE**,**0**)	√78	≈8.832

From the table above we can extract the currently preferred analyses: they are those analyses with the least barycentric shift, which in this case is √8. The possible spatial configurations are the following:

H_n	*it*	*bit*	*New Barycentre*
H_1	$\mathbf{00i_1t_1}$	$\mathbf{b_10i_1t_1}$	½(**DF**,**0**,2**B**2**G**,2**C**2**E**)
H_2	$\mathbf{0i_10t_1}$	$\mathbf{b_1i_10t_1}$	½(**DF**,2**B**2**G**,**0**,2**C**2**E**)
H_3	$\mathbf{0i_1t_10}$	$\mathbf{b_1i_1t_10}$	½(**DF**,2**B**2**G**,2**C**2**E**,**0**)

With one additional form we can seed the rest of the process: assume we encounter the form brɪtʰ *Brit*. The new acoustic phenomenon r we can assume corresponds to the model R, and hence interprets the phonological category **A**. We symbolise the phonological category $\mathbf{r_1}$. Note that we do not have any ambiguity. The only possible phonological analysis is $\mathbf{b_1r_1i_1t_1}$. How does this form change the shape of our configuration? We can calculate the barycentric divergence of $\mathbf{b_1r_1i_1t_1}$ for each of the H_n systems above:

H_n	*Barycentre*	*Measure of divergence*	
H_1	½(**DF**,**0**,2**B**2**G**,2**C**2**E**)	√19	≈4.359
H_2	½(**DF**,2**B**2**G**,**0**,2**C**2**E**)	√143	≈11.96
H_3	½(**DF**,2**B**2**G**,2**C**2**E**,**0**)	√218	≈14.76

Clearly, the least disruptive analysis is H_1, which gives us the following as the current configuration:

Brit	*it*	*bit*	*Barycentre*
$\mathbf{b_1r_1i_1t_1}$	$\mathbf{00i_1t_1}$	$\mathbf{b_10i_1t_1}$	⅓(2**D**2**F**,**A**,3**B**3**G**,3**C**3**E**)

Now watch what happens when we try to learn lɪtʰ. The new phonological category is given by the model RT, so let us set $\mathbf{l_1} = \mathbf{AE}$. The candidate analyses are $\mathbf{0l_1i_1t_1}$, $\mathbf{l_10i_1t_1}$, $\mathbf{l_1i_10t_1}$, $\mathbf{l_1i_1t_10}$. The barycentric divergences are shown in the following table. The preferred analysis is marked ⋆.

	lit	*H_1 Barycentric Divergence*	
⋆	$\mathbf{0l_1i_1t_1}$	⅓√139	≈3.930
	$\mathbf{l_10i_1t_1}$	⅓√157	≈4.177
	$\mathbf{l_1i_10t_1}$	⅓√1273	≈11.89
	$\mathbf{l_1i_1t_10}$	⅓√1939	≈14.68

As we can see, the algorithm quickly starts to reinforce the local barycentres, and hence to create the impression of particular 'inventories' at different positions in the linear order.

A parallel acquisitional and analytical process is the diagnosis of weirdness in unattested forms. Even at this early stage in our example, we can already discern reactions to, and rankings of, unattested forms. Again, we do not need a rule component, or 'grammaticality'.

Let us calculate the threshold of acceptability of this new system. With the acquisition of the form $\mathbf{0l_1i_1t_1}$, the location of the new barycentre shifts to ¼(2**D**2**F**,2**AE**,4**B**4**G**,4**C**4**E**), giving the following barycentric distances and divergence of all (four) attested forms:

Form	*Barycentric distances*	*Barycentric Divergence*	
$\mathbf{00i_1t_1}$	¼(2**D**2**F**,2**AE**,**0**,**0**)	$\frac{1}{4}\sqrt{117}$	≈ 2.704
$\mathbf{b_1r_1i_1t_1}$	¼(–2**D**–2**F**,–2**AE**,**0**,**0**)	$\frac{1}{4}\sqrt{117}$	≈ 2.704
$\mathbf{b_10i_1t_1}$	¼(–2**D**–2**F**,2**AE**,**0**,**0**)	$\frac{1}{4}\sqrt{117}$	≈ 2.704
$\mathbf{0l_1i_1t_1}$	¼(2**D**2**F**,–2**A**–3**E**,**0**,**0**)	$\frac{1}{4}\sqrt{253}$	≈ 3.976

The threshold of acceptability for this system is the maximum attested barycentric divergence, hence we have $\rho \approx 3.976$. This automatically gives us that the 'nonsense' (in the literal sense: having no attested meaning) form $\mathbf{b_1l_1i_1t_1}$ is non-weird (same divergence as $\mathbf{0l_1i_1t_1}$), whereas, for example, the nonsense forms $\mathbf{l_1b_1i_1t_1}$ and $\mathbf{r_1b_1i_1t_1}$ are both weird:

Form	*Barycentric distances*	*Bary'c Divergence*	
$\mathbf{b_1l_1i_1t_1}$	¼(–2**D**–2**F**,–2**A**–3**E**,**0**,**0**)	$\frac{1}{4}\sqrt{253}$	≈ 3.976
$\mathbf{l_1b_1i_1t_1}$	¼(–4**A**2**D**–4**E**2**F**,2**A**–4**DE**–4**F**,**0**,**0**)	$\frac{1}{4}\sqrt{1125}$	≈ 8.385
$\mathbf{r_1b_1i_1t_1}$	¼(–4**A**2**D**2**F**,2**A**–4**DE**–4**F**,**0**,**0**)	$\frac{1}{4}\sqrt{1013}$	≈ 7.957

Notice how the illusion of 'rule-based' behaviour emerges; we have not had to develop a theory of 'phonotactic constraints', yet the acquired knowledge of this simple system 'prefers' certain configurations to others.

2.8.2 *A Complete Estimate of the LE Barycentric Curve*

In this final portion of the chapter we provide a schematic representation of the likely outcome of a full barycentric analysis on a large corpus of acquisitional data. In the absence of a computational implementation of the method we have resorted to a good deal of logical reduction to anticipate the likely trajectory of the barycentric shifts, with an occasional heads-or-tails guess when there seemed to be no logical way to decide between competing analyses.

The purpose of this exegesis is to show that the key-length stipulation of four positions (§2.5.2) is quite accommodating, even of a language like LE; and also to aid the discussion of morphological structure in Chapter Three.

We begin with the first position, $t=1$. Building on the barycentric curve already established in the above discussion, we assume that further exposure to acquirable data will reinforce the position of the local barycentre away from phonological categories containing **B** ('vowels').

Now, in order to ensure that the categories we encounter really are found at $t=1$, we consider, as far as possible, forms such as $\mathbf{b}_1\mathbf{r}_1\mathbf{i}_1\mathbf{t}_1$, which have only one candidate parse. One interesting result is that the forms st, sk, and sp must be analysed as the realisations of single phonological categories, because of forms like strɪpʰ, splɪtʰ and skrapʰ. Further, because of forms like snɪpʰ and smakʰ, we also assume that the 'nasals' will tend to get parsed into the second packet. Because also of the preponderance of models containing H in the first packet, we are obliged to assume that **h** too is generally parsed into the first packet. Similarly the assignment of ʤ to the first packet would seem to be warranted because of the quantity of other models containing RD so parsed.

Table (2.29). Phonological Categories at $t=1$

Data	*Models*	*Categories*	*Data*	*Models*	*Categories*
drɪpʰ	DRT	**ADE**	brɪtʰ	DPR	**ADF**
grɪpʰ	DJR	**ADG**	ʤɪpʰ	DPRT	**ADEF**
tʰrɪpʰ	HT	**CE**	pʰrɪkʰ	HP	**CF**
kʰrɪkʰ	HJ	**CG**	ʧʰɪpʰ	HPT	**CEF**
hɪtʰ	H	**C**	sɪtʰ	DHR	**ACD**
strɪpʰ	DHRT	**ACDE**	splɪtʰ	DHPR	**ACDF**
skrapʰ	DHJR	**ACDG**	θrɛtʰ	HRT	**ACE**
ʃlɛpʰ	HPRT	**ACEF**	flɪtʰ	HPR	**ACF**
ðatʰ	DHT	**CDE**	ʒakʰ	DHPT	**CDEF**
vlad	DHP	**CDF**	zɪtʰ	DR	**AD**

From the table above we can construct the following table of phonological categories non-weird in LE at $t=1$. Cells in the table marked with a point ('.') represent locally weird points in the local phonological space at $t=1$. We note in passing that there are isolated forms like sʧuːpʰɪd, which may affect the barycentric curve, although in comparison with the inertia of the system as a whole, their contribution must surely be tiny. The form is attested, however, so we are obliged to include it nevertheless.

Table (2.30). Local Phonological Space at $t=1$

	0	**E**	**EF**	**F**	**G**	**EG**	**EFG**	**FG**
0		.	.	.	.	.	.	.
CD	.	ð	ʒ	v	.	.	.	.
C	h	tʰ	ʧʰ	pʰ	kʰ	.	.	.
AD	z	d	ʤ	b	g	.	.	.
ACD	s	st	sʧ	sp	sk	.	.	.
AC	.	θ	ʃ	f	.	.	.	.

Moving on to t=2, using exactly the same methodology, we discover the following. Note that we also include as data marginal forms here whose effect on the barycentre is probably very limited: **tʰsɛtʰsiː** (*tsetse*), and the names of the Greek letters **pʰsaj** (ψ) and **kʰsaj** (ξ). They are all attested forms, so we are not entitled to brush them aside. Given the paucity of these **s**-forms, though, we can assume that their mass is tiny in comparison with the mass of the **r**-/**l**-forms, and hence will be further from the barycentre than these latter forms, and thus be judged 'weirder' than them.

Table (2.31). Phonological Categories at t=2

Data	*Models*	*Categories*	*Data*	*Models*	*Categories*
bɪtʰ	0	**0**	brɪtʰ	R	**A**
blɪpʰ	RT	**AE**	kʰwɪtʰ	PR	**AF**
kʰjuːtʰ	JR	**AG**	sfiːə	HPR	**ACF**
snɪpʰ	DT	**DE**	smakʰ	DP	**DF**
kʰsəj	DHR	**ACD**			

Table (2.32). Local Phonological Space at t=2

	0	**E**	**EF**	**F**	**G**	**EG**	**EFG**	**FG**
0		.	.	.	.	.	.	.
D	.	n	.	m	.	.	.	.
A	r	l	.	w	j	.	.	
AC	s	.	.	f	.	.	.	.

Moving to t=3 we find something perhaps unexpected. We saw in §2.7.3 the analysis of **glɪntʰ**, which forced the nasal to be analysed as belonging to the third packet, and hence part of the interpretation of the phonological category at t=3. Consideration of data like **lɪɫtʰ** and **mɪɫkʰ** forces us to a similar conclusion about these instances of **ɫ**. That is, it is a part of the packet-three 'vowel': a species of 'postnasalisation', as it were. Hence we assume its model content includes D (in common with the postnasalised vowels) and H ('more high frequency energy').[22]

Further, data like **skrawʀʤ** force us to analyse the 'diphthongs' as packet-three vowels, stretched over the ASR-envelope.

Table (2.33). Phonological Categories at $t=3$

Data	*Models*	*Categories*	*Data*	*Models*	*Categories*
tʰrakʰ	A	**B**	tʰrʌkʰ	AT	**BE**
tʰrɔtʰ	APT	**BEF**	bʊkʰ	AP	**BF**
tʰrɪpʰ	AJ	**BG**	tʰrɛkʰ	AJT	**BEG**
pʰraŋkʰ	AD	**BD**	tʰrʌŋkʰ	ADT	**BDE**
kʰlɔmpʰ	ADPT	**BDEF**	drɪŋkʰ	ADJ	**BDG**
brɛntʰ	ADJT	**BDEG**	kʰlɑːkʰ	AR	**AB**
bɜːpʰ	ART	**ABE**	blʌwtʰ	ARPT	**ABEF**
kʰrawd	APR	**ABF**	tʰrajtʰ	AJR	**ABG**
pʰlɛjtʰ	AJRT	**ABEG**	pʰlɑːntʰ	ADR	**ABD**
bɜːntʰ	ADRT	**ABDE**	wʌwntʰ	ADRPT	**ABDEF**
mawntʰ	ADPR	**ABDF**	pʰajntʰ	ADRJ	**ABDG**
pʰɛjntʰ	ADJRT	**ABDEG**	pʰɔjntʰ	ADJPRT	**ABDEFG**
kʰwɔjtʰ	AJPRT	**ABEFG**	broːd	ARPT	**BCEF**
bruːd	APR	**BCF**	briːd	ARJ	**BCG**
blɛː	AJRT	**BCEG**	broːn	ADRPT	**BCDEF**
kʰruːn	ADPR	**BCDF**	pʰriːn	ADRJ	**BCDG**
kʰɛːn	ADJRT	**BCDEG**	tʰaɫkʰ	ADHR	**ABCD**
bʌɫkʰ	ADHRT	**ABCDE**	bɔɫtʰ	ADHRPT	**ABCDEF**
wʊɫf	ADHPR	**ABCDF**	tʰɪɫtʰ	ADHJR	**ABCDG**
pʰɛɫtʰ	ADHJRT	**ABCDEG**			

We note that the isolated form **kɪɫn** (*kiln*) poses something of a conundrum, since the barycentric shift analysis will try to parse the **n** into the third packet. Let us accept the theory's analysis and make this parse. We choose to analyse **ɪɫn** as the interpretation of a single phonological category: **ABCDEFG**, the most complex category we know. The model distribution over the ASR-envelope we can assume is (AJ,ADHRPT,D). We expect (and hope) that a more sensitive acoustic analysis will indeed reveal differences between the **ɫ** of *kiln* and the **ɫ** of *kilt*.

Table (2.34). Local Phonological Space at $t=3$

	0	**E**	**EF**	**F**	**G**	**EG**	**EFG**	**FG**
B	a	ʌ	ɔ	ʊ	ɪ	ɛ	.	.
BD	a^{n}	ʌ^{n}	ɔ^{n}	.	ɪ^{n}	ɛ^{n}	.	.
BCD	.	.	oː^{n}	uː^{n}	iː^{n}	ɛː^{n}	.	.
BC	.	.	oː	uː	iː	ɛː	.	.
AB	ɑː	ɜː	ʌw	aw	aj	ɛj	ɔj	.
ABD	ɑː^{n}	ɜː^{n}	ʌw^{n}	aw^{n}	aj^{n}	ɛj^{n}	ɔj^{n}	.
ABCD	aɫ	ʌɫ	ɔɫ	ʊɫ	ɪɫ	ɛɫ	ɪɫn	.

Finally we move to *t*=4, where we additionally note a couple of marginal forms which end in χ: lɔχ (*loch*), tʰɛχ (TEX).

TABLE (2.35). PHONOLOGICAL CATEGORIES AT *t*=4

Data	*Models*	*Categories*	*Data*	*Models*	*Categories*
kʰɪd	DRT	**ADE**	kʰab	DPR	**ADF**
kʰɔg	DJR	**ADG**	mɪʤ	DPRT	**ADEF**
kʰɪtʰ	HT	**CE**	kʰɪpʰ	HP	**CF**
kʰɪkʰ	HJ	**CG**	pʰɪʧ	HPT	**CEF**
lɔχ	RH	**AC**	kʰɪs	DHR	**ACD**
rɪst	DHRT	**ACDE**	lɪsp	DHPR	**ACDF**
rɪsk	DHJR	**ACDG**	kʰɪθ	HRT	**ACE**
pʰʊʃ	HPRT	**ACEF**	kʰɔf	HPR	**ACF**
wɪð	DHT	**CDE**	luːʒ	DHPT	**CDEF**
brɛjv	DHP	**CDF**	fɪz	DR	**AD**
rajm	DP	**DF**	brɪŋ	DR	**DJ**

TABLE (2.36). LOCAL PHONOLOGICAL SPACE AT *t*=4

	0	**E**	**EF**	**F**	**G**	**EG**	**EFG**	**FG**
0		.	.	.	.	.	.	.
D	.	.	.	m	ŋ	.	.	.
CD	.	ð	ʒ	v	.	.	.	.
C	.	tʰ	ʧʰ	pʰ	kʰ	.	.	.
AD	z	d	ʤ	b	g	.	.	.
ACD	s	st	sʧ	sp	sk	.	.	.
AC	χ	θ	ʃ	f	.	.	.	.

Note finally that we expect the barycentric method and/or the semantic consistency check to favour analyses of final n in LE which parse the D model into the packet at *t*=3 (that is, it is the 'nasality' of a 'nasal vowel' realised in the R-phase of packet 3), since we find that LE forms ending in n generally undergo 'nasal place assimilation' (*q.v.* §2.6.4).

⋆ ⋆ ⋆

The calculation of barycentric curves by the barycentric shift method could be undertaken with the help of a computational implementation. The creation of suitable training databases is also essential. Both tasks are not trivial and demand a separate study of their own.

However, we believe that the preceding presentation more than justifies future investment in such a project, as it shows that the system is theoretically capable of acquiring quasi-systematic behaviour from unstructured learning stimuli. For the purposes of the present study we are confident that we have demonstrated the credibility of the hypothesis that the phonology is simply a structured set of integers (hash keys), with no rule component and no notion of derivation or other transformational devices.

2.9 *Notes to Chapter Two*

1. 'Long into the night, all the wine drunk, the conversation turned to phonology.' This rarely quoted gem comes from an otherwise oft-quoted passage in the preface to the medieval Chinese *Guangyun* dictionary of 1008 [大宋重修廣韵、序二].
2. The study of how human beings actually come by the m_i is undertaken in Chapter Three. We show there that it is possible to define an algorithm which semi-automatically assigns morphosyntactic meanings to phonological forms using only the notion of 'distance' between phonological forms. In §2.3 below we show that a notion of distance is available independently of the requirements for a theory of language acquisition, and hence that a theory of language acquisition is available within the metatheory proposed here.

It is perhaps worth underlining that the theories expressible within this metatheory are also capable of analysing the continued acquisition of linguistic forms throughout the lifetime of the speaker, for which see §2.3.2.
3. Or derivations: 'rules' for our purposes are any functions which partition Φ, as discussed in §2.1.3.
4. Following Kaye (1996), it seems reasonable also to assume that individuals have different interpretation functions (we are individually identifiable by our speaking voice), and that recognisable social groups are marked by broader band similarities in the phonetic interpretation functions of their members ('accent').

Our ability to process unfamiliar forms demonstrably depends on the familiarity of the interpretation function used to utter it. Even attested forms can become hard to process when delivered through a particularly unfamiliar interpretation function ('foreign accent'). Conversely, any unattested form can be rendered more 'native sounding' by processing it through a familiar interpretation function ('saying it in a native accent').

These observations are, however, independent of the notion of affiliation explored in the remainder of the chapter. That notion of affiliation is based purely on phonological form, *modulo* any phonetic interpretation.
5. Another significant loss would appear to be in the role phonological rules play in parsing. However, it is quite easy to build robustness into the parsing system using exactly the same device that is used to diagnose affiliation: whereas the bare bones of the parser are provided, unsurprisingly, and in common with other theories, from the definition of phonological representations (*e.g.* Williams 1997). Roughly, the definition of phonological representations provides the parser with candidates (the 'recogniser'); the device used to diagnose affiliation is used as a metric to select (or perhaps rank) these candidates. In other words we parse so as to minimise 'weirdness', and maximise 'familiarity'. This topic is discussed further in §2.7.
6. An argument that is nearly universally used as supporting evidence for rules is that they allow less information to be stored in the lexicon (*e.g.* Bromberger & Halle 1989).

The refutation of the idea that lexical resources are 'at a premium' (*ibid.*) is presented in considerable detail in Chapter One.

7. See further Chapter Three for ways in which this distance metric can be exploited to describe morphological behaviour, and the acquisition of an individual's particular m_i.

8. We may further speculate that there is a time-dependent bias to the mass, based on recent frequency of occurrence of a form; we shall ignore this fine tuning for the purposes of this chapter. Again, we return to the topic in Chapter Three.

9. One thinks of the Norman invasion of England in 1066AD, and the concomitant effects on the characterisation of 'English sounding' words.

10. Another area where barycentric distance may play a key role is phonological parsing, a topic addressed in §2.7. Acquisition is discussed further in §§2.7–8.

11. See for example Hey 1953.

12. An exhaustive study of this toy phonological system requires 256 tables like the one above. Space constraints prohibit their inclusion in this study, but their calculation can be safely left as an 'exercise for the reader'.

Additionally, we can use analysis of the mathematics of the system to prove some general results, as for example in a two-word language where the two words occupy opposite apexes of the cube (that is, the two forms are the furthest possible distance apart), then all phonological forms are equidistant from the barycentre. Thus, for a speaker of the two word language {**A**} and {**I**,**U**}, all forms are equally 'weird/acceptable' (the barycentric distance of all forms is $\sqrt{3/4}$, approximately 0.866).

13. The choice of symbols for the Categories is arbitrary. They have been chosen deliberately to avoid confusion with the 'categories' of existing theories, and they have been chosen deliberately to obscure the relationship with their acoustic interpretation (§2.2). This is to underline the fact that the phonology is *not* modality specific, and has no special relationship with acoustic objects.

14. Significantly, Goh 1996 gives Mandarin Chinese a templatic analysis.

15. Given a template of length α, a phonological string of length less than α cannot be parsed. A phonological string of length equal to α has exactly one parse. For a longer string, if the first α components ($c_1, \ldots c_\alpha$) of the phonological string are not accepted by the template (first parse), then the components ($c_2, \ldots c_{\alpha+1}$) are checked (second parse); if this substring is not accepted by the template, the next substring ($c_3, \ldots c_{\alpha+2}$) is checked (third parse). If this process continues to the α-th parse, checking substring ($c_\alpha, \ldots c_{2\alpha-1}$), and if this parse fails, then the substring ($c_1, \ldots c_\alpha$) contains no parsable components. Thus it requires at most α parses to process the first (and by induction, any) substring of length α.

16. Other changes may also occur, depending on an individual's tolerance to barycentric distance. We might suppose that some (conservative) individuals are unwilling to allow attestations that are too far removed from their barycentres; other speakers, the author included, are extremely tolerant of barycentric distance. The strategies available to conservative speakers include ignoring the new form altogether or arbitrarily selecting a closely related form (in the sense of Chapter Three) which is nearer to the barycentre.

17. Chomsky assumes that if human beings could communicate by 'telepathy', then 'there would be no need for a phonological component' (Chomsky 1995:221). Rather, there would be no need for a *phonetic interpretation function*. There would still be a need for a 'telepathic' interpretation function for phonological objects.

18. It follows from our definition of A and R that on the continuum between *vowel* and *approximant* lies the distinction between tense and lax vowels. We shall use the model AR to characterise the tense vowels, and A to characterise the lax vowels of South Eastern British English in §2.7.3 below.

19. This entails, by the way, that English has no contrastive nasalised vowels, which is true, and that the nasalised vowels of other languages should be 'Anglicised' to vowels followed by a delayed N model (an 'assimilating nasal'), which is also true.
20. In fact, the author has still to be convinced that NPA takes place in words like *songbird* and *sometimes*. For him, at least, it only seems to be post-vocalic '*n*' which undergoes NPA.
21. The details of the author's phonetic interpretation function are in close agreement with the description of London English found in Wells 1982 (§4.2:301–34), in particular the variety described as 'London Regional Standard' (p.303 *et passim*).
22. This ɫ does not appear to be the result of envelope-overlap in the same way that pre-consonatal n seems to be. Therefore we must assume that the ɫ-portion is realised in the S-phase, and the vowel portion in the A-phase.

Chapter Three

Morphology

> I should not wonder if *morro*, *manro*, and *panis* were connected, perhaps derived from the same root; but what is that root? I don't know—I wish I did; though, perhaps, I should not be the happier.
>
> — *Borrow* 1851:267

It has been a commonplace observation in morphology that word-forms in any given language can be 'related' in some way. It is then generally the task of the morphologist to turn this intuition into a testable theory. There has been no shortage of proposals both from morphologists (a decent snapshot is given in Spencer 1991) and increasingly from phonologists (from Lexical Phonology, Hargus & Kaisse 1993, to Government Phonology, Kaye 1995a). In this chapter we attempt to formalise and then explore how word-forms can be related, and indeed *why* word forms should be related at all, building on the notions introduced in the previous two chapters.

Given the equation established in Chapter One between secondary hashkeys and phonologically bound forms, we consider this an appropriate place in which to provide a more detailed account of secondary hashing as we find it in human language. Morphology being traditionally the domain of discussion of bound forms, we use this chapter to explore the fine-structure of secondary hashing and its interpretation, in both the acoustic and hashkey semantics of Chapter Two.

We explore in the later sections of this chapter the idea that relatedness is an essential ingredient in the acquisitional process that allows the automatic population of the lexicon. The system we propose is completely general, and works by extracting morphosyntactic information from the learning stimulus and creating a fairly simpleminded generalisation, which echoes the more traditional type of 'word-formation rule'. We illustrate the approach with a detailed case study, and provide some summary speculations for avenues of research suggested by this study. The culmination of this exercise is the demonstration of the processes and conditions needed to acquire morphological knowledge that has traditionally been described as rule-based (such as knowledge of 'paradigms', 'regular' and 'irregular' forms and even wilful 'mistakes' like jocular *thunk* for the past tense of *think*).

These fundamentals pave the way for understanding the central theme of this work: an understanding of the relationship between syntactic structures and phonological structures, which is the topic of Chapter Four.

3.1 *PWords*

We assume that the long recognised division by definition of phonological forms into *bound* and *free* is valid (*q.v.* Chapter One), and that bound forms in turn occur in two kinds, *prefixes* and *suffixes*. It is our contention, asserted in Chapter One, that the secondary hashkeys of the human language lexicon are encoded into a phonological representation precisely as bound forms.

3.1.1 *The Structure of Secondary Hashkeys*

How are we to define the phonological properties of bound forms? Clearly they cannot consist of a sequence of four or more phonological categories, otherwise they would constitute at least an additional phonological key. They are therefore representations consisting of a sequence of less than four phonological categories. By simple observation it is clear that the sequence must also be a sequence of more than one phonological category: in LE for example there are sequences which clearly have suffix-like properties, but which require more than one phonological category to represent. A clear case is the suffix *-man* of *postman*. This suffix has no independent existence (it always needs a 'host'), and it needs at least two phonological categories to account for its realisation as mən—(**ADEF**,**ADEG**).

Bound-form length should therefore be greater than one and less than four. LE doesn't appear to have any clear-cut cases of a bound form requiring three categories. Candidates like *-hood* (in *falsehood*) can all typically be analysed as phonological keys in their own right (attested in LE, where *hood* is a type of headgear). Further, complexes like *-ingly* (as in *swimmingly*) can either be analysed as integral keys (there is no attested word *ingly* in LE, but there is a name *Bingly*), or as a sequence of length-two bound forms *-ing-ly* (discussed further below in §3.2).

For these reasons we stipulate, universally, a strict *two category* template for bound forms (Axiom 3.1). As with the universal key size of four (Axiom 2.15), this may turn out to be wrong; bound forms may need three categories. The actual choice does not affect the structure of the theory, and ultimately depends on the allocation of computational resources to the hash system (§2.1.4).

Axiom (3.1). Bound Forms

A bound form *is a sequence of* β *phonological categories,* $0<\beta<\alpha$.
For Natural Language, $\alpha=4, \beta=2$.

In the next section we define how bound forms and phonological keys combine to form *phonological words*, and how these objects can be strung together to form 'connected speech'. We also spend some time discussing an appropriate parsing mechanism.

3.1.2 *Phonological Words (PWords)*

In this subsection we introduce the notion of a *phonological word*, or *PWord*. Intuitively, a PWord is a phonological key, or a phonological key with attached bound forms. To capture this, we again invoke sequences and define a PWord to be a sequence of zero or more *prefixes* followed by a key followed by zero or more *suffixes*.

Axiom (3.2). The PWord

Let $\mathbf{K}=\wp(\kappa)$ be the set of all phonological categories. Let $\mathbf{K}^{\beta}$ be the set of all bound forms, and let $\mathbf{K}^{\alpha}$ be the set of all keys. A *PWord* π is a member of $(\mathbf{K}^{\beta}\times\ldots\mathbf{K}^{\beta}\times)\mathbf{K}^{\alpha}(\times\mathbf{K}^{\beta}\times\ldots\mathbf{K}^{\beta})$. Equivalently $\pi\in(\mathbf{K}^{\beta})^{n}\times\mathbf{K}^{\alpha}\times(\mathbf{K}^{\beta})^{m}$, for $n,m\geq 0$.

With this enriched structure, we need to consider the question of phonetic interpretation. Much depends on to what extent we believe that the PWord structure is interpreted (by the phonetic interpretation function, or other interpretation functions) over and above the interpretation of the phonological categories that make it up. If we maintain our commitment to the idea that interpretation functions are Tarskian semantics, then we should expect that all aspects of the object language (here, the phonology) are interpreted.

PWord structure depends crucially on the distinction between bound forms and free forms. We shall assume therefore that this distinction is also interpreted. We assume, in other words, that PWord structure is *not* underdetermined by the interpretation functions (like the acoustic signal).

We note, however, that any interpretive device used to encode the difference between bound and free forms cannot by definition come from the repertoire of acoustic models as already given in Chapter Two. In the case of LE, and many other languages, the difference is encoded by the phonetic interpretation function as a difference in 'stress'. Perhaps we should more correctly talk of a difference in the aggregate energies of the acoustic models across bound and free forms. Thus free forms are readily recoverable for LE speakers, since they are pronounced 'more energetically' than free forms.

We suppose that this is in general true, and that even for languages which are claimed not to employ ‘stress’, something plausibly akin to an energy differential can be found in the signal. We accordingly introduce an additional symbol into the calculus of acoustic models, such that a model bearing this symbol is understood to be realised with ‘less energy’ than the corresponding model without it.

Definition (3.3). ‘Unstressed’ Models

For any acoustic model M*, there is an* unstressed acoustic model M^0.

Any sequence of unstressed acoustic models $(M^0,N^0,\ldots)$ *can be symbolised* $(M,N,\ldots)^0$.

An equivalent formulation of this idea is that for any primary hashkey *k* and for any two secondary hashkeys *s* and *t*, the interpretation of *k* cannot be *exactly* the same as the interpretation of *s* followed by the interpretation of *t*. That is, $[k]\neq([s],[t])$. This follows directly from the requirements of a Tarskian semantics, since it ensures that the phonological distinction between *k* and *(s,t)* is not destroyed, thus preserving the isomorphism.

The interpretation of a PWord follows straightforwardly:

Definition (3.4). Interpretation of PWords

The interpretation of a PWord $\pi=(\xi_1,\ldots,\xi_n,\delta,\zeta_1,\ldots,\zeta_m)$, *written* $[\pi]$, *is given by* $[\pi]=(([\xi_1]^0,\ldots,[\xi_n]^0),[\delta],([\zeta_1]^0,\ldots,[\zeta_m]^0))$.

Thus we can illustrate the interpretation with an example PWord in (3.5), using the LE interpretation function to get ɪmˈpʰɔsɪbʊɫ, a form we might recognise as *impossible*.

Example (3.5). ‘Impossible’

t=	(1	2)	(1	2	3	4)	(1	2)	(1	2)
	Affix		*Primary Key*				*Affix*		*Affix*	
Categories	**BDG**	**0**	**CF**	**0**	**BEF**	**ACD**	**BG**	**ADF**	**ABCDF**	**0**
Models	ADJ^0	0	HP	0	APT	DHR	AJ^0	DPR^0	ADHPR^0	0
LE	ɪⁿ	ˈ	pʰ		ɔ	s	ɪ	b	ʊɫ	

In the act of communicating, then, a speaker assembles all the instructions necessary to make the correct lexical accesses to find the structures she wishes to communicate. These instructions comprise by definition PWords (primary and secondary hashkeys). Each PWord is delivered to the interpretation function, where it is interpreted, and thus communicated. ‘Connected Speech', then, is nothing more than the phonetic interpretation of a string of PWords, which we explore in more detail in Chapter Four.

3.1.3 *Phonological Parsing*

Having established a reasonable interpretation function for LE (§2.8), we can see at once why a key length of less than four phonological categories does not adequately capture the intuition that an utterance like ˈskrawndʒ *scrounge* is an 'unanalysable word'. From the tables given in Chapter Two, we can see that this key requires exactly four phonological categories (3.6).

Example (3.6). 'Scrounge'

t=		(1	2	3	4)
Categories		**ACDG**	**A**	**ABDF**	**ADEF**
Models		DHJR	R	ADPR	DPRT
L.E.	ˈ	sk	r	aw^{n}	dʒ

Words in LE which are apparently longer are always 'analysable', in that speakers quite naturally 'find' a smaller word inside the larger, and are able to use this smaller word for creating related words, often 'nicknames'. For example it is quite natural to imagine that a notoriously scruffy individual whose appearance is often *crumpled* should be given the nickname *Crumper* (or *Crumpers*, or, if he were from the South West of England, *Crumpy*, or from Australia, *Crumpo*). A nickname like *Crumpler* does not have the same natural 'ring' to it. We can understand this from the four-category restriction quite easily. The form *crumple* requires six categories (3.7). Our phonology has to parse this minimally into a four-category (primary) key ˈk^{h}rʌmp^{h} plus a residue ʊɫ (note that this latter residue also appears in the parsing of *impossible* (3.5)). Hence the intuition that *crumple* is 'analysable'.

Example (3.7). 'Crumple'

t=		(1	2	3	4)	(1	2)
Categories		**CG**	**A**	**BDE**	**CF**	**ABCDF**	**0**
Models		HJ	R	ADT	HP	ADHPR^{0}	0
L.E.	ˈ	k^{h}	r	ʌ^{n}	p^{h}	ʊɫ	

We shall continue to maintain the hypothesis that all primary hashkeys in human languages are four categories in length. Where words apparently exceed this limit, we simply let the phonology parse the string into a primary key, plus 'dummy' (or Cranberry) morphology (Spencer 1991).

We turn in the next section to an analysis of how these structures are interpreted as instructions to access the lexicon, and hence how they are used to recover their 'meanings'.

3.2 *Hashkey interpretation of PWords*

We turn in this section to the hashkey interpretation of these larger structures. A semantics has to be found which makes sense of PWord structure in terms of primary and secondary hashing. We begin with some general introductory material on the structure of lexical space, which follows from the definitions of Chapter One, and we introduce some notational conventions and concepts which build on the formal system of Chapter Two. We then go on to examine various patterns of lexical access observable in language use, and use these observations to develop a theory of how PWords encode instructions for the lookup mechanisms.

3.2.1 *The Structure of Lexical Space*

The phonology defines the interface between the lexicon and other cognitive faculties through an $\mathfrak{H}(3)$ hashtable. Our investigations led us to propose that the human language hashing system was based on a categorial algebra with seven primitives, expressions from which exist in a 6-dimensional array (four positions for the primary hashkey, two for the secondary hashkey). The topology of the $\mathfrak{H}(3)$-space, then, has the property that it consists of a $6\times4=24$ dimensional space with a 'bolted on' $6\times2=12$ dimensional space.

The 'bolting on' is captured mathematically by taking the Cartesian product of the two spaces, thus any point in this space can be uniquely determined by two vector specifications: a position vector in primary hashkey space, and a position vector in the respective secondary hashkey space. That is, any point $\mathbf{Q}$ in the hashtable is the structure $(\mathbf{q}_1,\mathbf{q}_2)$, where $\mathbf{q}_1$ is a primary hashkey vector quantity, and $\mathbf{q}_2$ is a secondary hashkey vector quantity. Phonological representations are interpreted as quantities in this hashtable space (Chapter One).

Definition (3.8). Hashtable Vectors

A hashtable vector *is the structure* $(\mathbf{q}_1,\mathbf{q}_2)$, *where* $\mathbf{q}_1$ *is a primary hashkey vector quantity, and* $\mathbf{q}_2$ *is a secondary hashkey vector quantity.*

Further, at each point in the hashtable is a list of the LNodes hashed to that point. Let the position of a particular LNode on a list of LNodes be designated by a vector. The position of any individual lexical item can now be uniquely specified as a pair ($\mathbf{Q}$,$\mathbf{n}$), where $\mathbf{Q}$ is a point in the hash space, and $\mathbf{n}$ is a vector representing the position of the LNode on the LNode list located at $\mathbf{Q}$. Equivalently, we can say that the location of any individual lexical record can be specified as the vector triple ($\mathbf{q}_1$,$\mathbf{q}_2$,$\mathbf{n}$). We shall also occasionally use the notation ($\mathbf{q}_1$,$\mathbf{q}_2$,#) for the point ($\mathbf{q}_1$,$\mathbf{q}_2$) in hashkey space, when we wish to emphasise the relationship between a hashkey and the list of records hashed to that key.

Definition (3.9). Vectors in Lexical Space

A vector in lexical space *is the structure* ($\mathbf{Q}$,$\mathbf{n}$), *where* $\mathbf{Q}$ *is a hashtable vector, and* $\mathbf{n}$ *is a vector representing the position of the LNode on the LNode list located at* $\mathbf{Q}$.

Our task, then, is to extend the definition of distance and related notions given in Chapter Two to this tri-partite lexical space. Defining positions in lexical and hashtable space using vector quantities means that we already have definitions of distance. We call this quantity the *PVector* between the two points in the hashtable.

Definition (3.10). PVectors

The PVector *between any two points* $\mathbf{Q}$ *and* $\mathbf{R}$, *where* $\mathbf{Q}=(\mathbf{q}_1,\mathbf{q}_2)$ *and* $\mathbf{R}=(\mathbf{r}_1,\mathbf{r}_2)$ *is given by* $\delta(\mathbf{Q},\mathbf{R})=_{df}(\mathbf{r}_1-\mathbf{q}_1,\mathbf{r}_2-\mathbf{q}_2)$.

The actual 'physical' distance between two LNodes $\lambda_1=(\mathbf{Q},\mathbf{q})$ and $\lambda_2=(\mathbf{R},\mathbf{r})$ can be given by $\delta(\lambda_1,\lambda_2)=_{df}(\delta(\mathbf{Q},\mathbf{R}),\mathbf{r}-\mathbf{q})$. Equivalently we have $\delta(\lambda_1,\lambda_2)=_{df}(\mathbf{r}_1-\mathbf{q}_1,\mathbf{r}_2-\mathbf{q}_2,\mathbf{r}-\mathbf{q})$.

Definition (3.11). Distances in Lexical Space

The distance between two LNodes $\lambda_1=(\mathbf{Q},\mathbf{q})$ *and* $\lambda_2=(\mathbf{R},\mathbf{r})$ *is given by* $\delta(\lambda_1,\lambda_2)=_{df}(\delta(\mathbf{Q},\mathbf{R}),\mathbf{r}-\mathbf{q})$.

All other vector operations, in particular summing and scaling, follow trivially from the vector characterisation of points in lexical space. However, there is an interesting twist to the story. Hashkey space is defined to be finite, so there is the very real possibility that the process of summing vectors leads to vectors which point outside the definitional limits of this space. We shall assume that such vectors are undefined with respect to lexical access.

3.2.2 Patterns of lexical access

As a starting point to a theory of lexical access through PWords, we note that a PWord contains exactly one phonological key, and we know that a phonological key is interpreted as a primary hashkey. We also know that a phonological key with no bound forms is a perfectly well-formed PWord, and can be used to access LObjects. So in some sense a PWord represents a hashtable location.

Take a phonological key, and PWord of English, say, ban *ban*, and let $\mathbf{b}_1$ be the vector which is the interpretation of *ban* in primary hashkey space. Note the absence of an overt bound form in the PWord; and note also the requirement for both a primary and a secondary hashkey to perform a lexical access. We take this as evidence that a phonological key does in fact encode a complete lookup, with some secondary hashkey (call it '$\mathbf{0}_2$') 'understood'. The interpretation of *ban* in lexical space is therefore the hashed LNode list ($\mathbf{b}_1$,$\mathbf{0}_2$,#), where we can access the LObjects [BAN,n.], [BAN,v.] and others. Given the acoustic interpretation of this structure as ban we must assume that the 'understood' secondary hashkey in this case is the phonological structure (**0**,**0**).

What happens when we process the same phonological key, but this time augmented by a bound form, *-ed* ? The PWord is band ((*ban*),*d*), and with this we can test our intuitions about which LObjects are accessed by processing it. On the one hand it looks as if we actually access *two* LObjects, namely the LObject [BAN,v.] and the LObject which inserts 'past tense'. Let us call this latter LObject [past], and let us assume that our syntactic calculus provides an operation of combination which allows us to combine LObjects such that we can state that [BAN,v.,past]=[BAN,v.]+[past] (*cf.* Chapter Four). We appear to be accessing the complex LObject [BAN,v.,past]=[BAN,v.]+[past].

On the other hand, the same PWord lets us access a separate (list of) LObjects, namely [BAND,v.], [BAND,n.], ... There seems, therefore, to be a bifurcation in our intuitions. The PWord ((*ban*),*d*) gives us (minimally) the following patterns of access (3.12).

(3.12) *Accessing* **((ban),d)**
Pattern 1 [BAN,v.,past]=[BAN,v.]+[past]
Pattern 2 [BAND,v.] or [BAND,n.]

Pattern 2 suggests that the *entire* PWord encodes a single lexical access, whereas Pattern 1 suggests it might encode two. We discuss Pattern 2 in a following section (§3.2.3), where we introduce the notion of a *distributed LObject*. For the moment we concentrate on pattern 1.

Given the assumption just made about the interpretation of a phonological key, we can see readily how the PWord ((*ban*),*d*) comes to encode two lexical accesses. The interpretation of the phonological key *ban* is, by definition ($\mathbf{b}_1$,$\mathbf{0}_2$,#); this represents one access. The interpretation of the suffix is by definition a secondary hashkey, call it $\mathbf{d}_2$. In order to make the second look-up, it has to be interpreted in conjunction with a primary hashkey. We already have one of those to hand (it is $\mathbf{b}_1$), so we can naturally assume that the second access point encoded is ($\mathbf{b}_1$,$\mathbf{d}_2$,#). The distribution of forms in the lexicon is therefore

(3.13) ***Lexical distribution implied by* ((ban),d)**

1^{ary} *Hashkey*	2^{ary} *Hashkey*	*LObjects*
$\mathbf{b}_1$	$\mathbf{0}_2$	[BAN,V.] …
$\mathbf{b}_1$	$\mathbf{d}_2$	[past] ...

Phonological forms which contain multiple bound forms can be accommodated in the same way. We can view sequences of affixes as encoding multiple look-ups from the same 'page' of lexical memory defined by the primary hashkey. Each such look-up uses on the average precisely half the computational resources of a full-blown double-hash look-up, so represents a particularly efficient way of accessing local areas of the lexicon. Let us see how this would work with a PWord *bannings*, which we shall assume has the phonological structure ((*ban*),*ing*,*s*).

Let $\mathbf{i}_2$ and $\mathbf{s}_2$ be the hashkey interpretations of *-ing* and *-s* respectively (they are secondary hashkeys). The structure ((*ban*),*ing*,*s*) immediately gives a sequence of three lexical accesses: ($\mathbf{b}_1$,$\mathbf{0}_2$,#), ($\mathbf{b}_1$,$\mathbf{i}_2$,#), and ($\mathbf{b}_1$,$\mathbf{s}_2$,#). Assume further that the LObjects accessed are, respectively [BAN,V.], [gerund], [plural]. Of course other LObjects may be available at these positions, but we assume that the three selected here have been made available by suitable ambiguity resolution mechanisms. The lexical distribution implied is

(3.14) ***Lexical distribution implied by* ((ban), d) *and* ((ban), ing, s)**

1^{ary} *Hashkey*	2^{ary} *Hashkey*	*LObjects*
$\mathbf{b}_1$	$\mathbf{0}_2$	[BAN,V.] …
$\mathbf{b}_1$	$\mathbf{d}_2$	[past] ...
$\mathbf{b}_1$	$\mathbf{s}_2$	[plural] …
$\mathbf{b}_1$	$\mathbf{i}_2$	[gerund] ...

We can therefore define the hashkey interpretation of a PWord formally. A complete implementation of a hashkey interpretation function is given in Appendix B:

Definition (3.15). Hashkey interpretation of PWords

The hashkey interpretation of a PWord *is a sequence of n points* $\mathbf{Q}[i]_{0<i\leq n}=(\mathbf{q}_1[i],\mathbf{q}_2[i])$ *in hashkey space, such that* $\mathbf{q}_1[i]=\mathbf{q}_1[j]$ *for all* $0<i,j\leq n$, *and there is a* z, $0<z\leq n$, such that $\mathbf{q}_2[z]=\mathbf{0}_2$.

In the next subsection we consider the details of generalising and implementing these multiple-access structures, and provide a mechanism for explaining the second pattern of access, which gives us the intuition that ((*ban*),*d*) can also be a single LObject [BAND,v.], [BAND,n.].

3.2.3 *Distributed LObjects*

Although we feel that the past tense *banned* is 'two words in one', there is still a residual feeling that it is also 'one word'. In the discussion above we anticipated one aspect of the syntactic calculus, and that was an operation of *combination*. This is not a controversial point, as it seems to be a minimal requirement of any syntactic theory that it is able to express the combination of syntactic objects (*q.v.* Chapter Four). Insofar as the combination of two syntactic objects is itself a syntactic object, we can capture the intuition that the two-words-in-one of *banned* are sort-of-one word. At the point of lexical access there are two LObjects associated with *banned*, but these LObjects are instantaneously syntactically combinable into the single LObject [BAN,v.,past], hence the sort-of-one word intuition.

Now consider *band*. According to our discussions above, this too encodes two lexical accesses. In order to turn two LObjects into the single LObject [BAND,n.], we simply have to ensure that our syntactic calculus has an identity LObject. Let us call it 0. One of the LObjects accessed by *band* we can assume therefore is just 0, which in a sense has no intrinsic content of its own (hence the intuition that 'it feels like it doesn't exist'). The syntax then combines [BAND,n.] with 0 to get [BAND,n.]=[BAND,n.]+0.

Postulate (3.16). Syntactic Identity

Any syntactic calculus with an operation of combination $\oplus$ *must contain an identity element with respect to* $\oplus$.

So we can see that the 'meaning' [BAND,n.] is *distributed* across two LObjects, just as the 'meaning' [BAN,v.,past] is.

A nice question at this juncture is which of the two LObjects of the distribution are stored where? Do we have ($\mathbf{b}_1$,$\mathbf{0}_2$,#) accessing [BAND,n.] followed by ($\mathbf{b}_1$,$\mathbf{d}_2$,#) accessing 0, or *vice versa*? A *reductio ad absurdum* provides a convincing answer in this case. Imagine that the order is indeed ($\mathbf{b}_1$,$\mathbf{0}_2$,#) accessing [BAND,n.] followed by ($\mathbf{b}_1$,$\mathbf{d}_2$,#) accessing 0. The presence of the LObject 0 on the list at ($\mathbf{b}_1$,$\mathbf{d}_2$,#) would seem to imply that *band* should be interpretable as [BAN,v.] followed by 0, and therefore 'means' exactly the same as *ban* does. However, it is clearly false that *band* is a circumlocution for *ban*.

So we are forced to accept the counterhypothesis that the accesses are: ($\mathbf{b}_1$,$\mathbf{0}_2$,#) accessing 0 followed by ($\mathbf{b}_1$,$\mathbf{d}_2$,#) accessing [BAND,v.]. We do need some mechanism, however, to prevent us sprinkling '*ban*' all over the place, since it should combine 'invisibly' with any syntactic object! We do need such a mechanism independently of this problem, however, as we discover in the rest of this and the following subsection.

By generalising this notion, we arrive at a characterisation of a special case of lexical access that ultimately recovers a single LObject, but uses several lexical accesses to do so. Counter-intuitive as this may sound at first, it seems to be minimally required for forms like *band*, and in general for so-called cranberry morphology. Since all wellformed sentences are by this definition 'distributed LObjects', the special case we need is that all the LObjects in a genuine distribution do not seem to create compositional *conceptual-semantic* structure (conceptual-semantic structure we assume is what distinguishes the LObject [DOG,n.] from the LObject [CAT,n.], for instance). On the other hand, they do seem to create compositional morpho-syntactic structure. In the vector-based notation we are assuming here, this idea translates into a criterion for distributed LObjects that not more than one member of the distribution can possess non-null conceptual-semantic structure.

Axiom (3.17) Distributed LObjects

A distributed LObject *is a sequence of LObjects* $L[1\ldots n]$ *where*

i. *there is a single LObject* $L^{+}=L[1]+\ldots L[n]$.

ii. *there is no more than one a* ($1\leq a\leq n$) *such that the conceptual-semantic content of* $L[a]$ *is non-null.*

We say that $L[1\ldots n]$*is* syntactically equivalent to L^{+}.

Without modification the same idea can even be used to characterise the lexical storage and retrieval of arbitrarily long idioms for single LObjects.

For example, *kick the bucket* is a sequence of several lexical accesses, which intuitively feel like a single LObject, [DIE,V.]. Thus we can say that *kick the bucket* encodes a distributed LObject, which evaluates to [DIE,V.]. Assuming the phonological analysis is (*kick*) (*the*,(*buck*),*et*) we might assume the following distributed LObject, where $\mathbf{k}_1$,$\mathbf{t}_2$,$\mathbf{u}_1$ and $\mathbf{e}_2$ are the hashkey interpretations of *kick*, *the-*, *buck* and *-et* respectively:

(3.18) ***Lexical distribution implied by* kick the bucket**

1*ary*	2*ary*	*LObjects*
$\mathbf{k}_1$	$\mathbf{0}_2$	[DIE,V.] ...
$\mathbf{u}_1$	$\mathbf{0}_2$	0 ...
$\mathbf{u}_1$	$\mathbf{e}_2$	0 ...
$\mathbf{u}_1$	$\mathbf{t}_2$	0 ...

Note, however, that the above lexical distribution implies that [DIE,V.] should be equally recoverable from just (*kick*). This doesn't seem to be entirely true, so we assume further, and not unreasonably, that the LObjects which make up a distributed LObject are 'physically' linked together in a chain-like configuration. For example, once we have processed an LObject *cran*, we are already expecting the LObjet *berry*; once we have processed *kick* (and decided that we are not interested in the other LObjects hashed to (*kick*), like [KICK,V.], [KICK,n.]) we are already expecting *the* expecting *buck* expecting *et*. With this mechanism we can understand why we can't just sprinkle *ban* (accessing 0) any old where: *band* is a distributed LObject, so we must assume that the LObject 0 at ($\mathbf{b}_1$,$\mathbf{0}_2$,$\boldsymbol{a}$), say, contains a 'physical link' to the LObject [BAND,V.] at ($\mathbf{b}_1$,$\mathbf{d}_2$,$\boldsymbol{b}$), say. Therefore, having encountered *ban*, and decided that it means 0, we are drawn inevitably along the physical link to ($\mathbf{b}_1$,$\mathbf{d}_2$,$\boldsymbol{b}$), expecting *-d*, and the LObject [BAND,V.].

We turn to a more rigorous investigation of these 'physical links' in the following subsection (§3.2.4).

3.2.4 *Chains*

So we need to assume that in addition to the LObject itself, the record stored at each LNode must include a 'pointer' to the next LObject in the distribution. Let us agree to call such a configuration a *chain*. In this way we can view every LObject as a distributed LObject: a single LObject distributed LObject simply points to some designated 'end-of-chain' object.

Thus we redefine the structure of the LObject record here as a pair {**L**,**N**}, where **L** is the morphosyntactic/semantic specification that we have been calling an 'LObject' thus far (*e.g.* 0 and [BAN,n.]), and **N** is a pointer to an LNode: a vector in lexical space. The LObject stored at **N** is the next LObject in the distribution. A chain has a definite start point and a definite endpoint. We identify the start point of a chain with a diacritic metasymbol ⋆, and we define a special vector ∞ that stands for a generic 'end-of-chain' pointer, with the definitive property that for any vector **v**, $\infty + \mathbf{v} =_{df} \infty$.

Definition (3.19). LObjects and Chains

An LObject *is a pair* $\{L,N\}$*, where L is a morphosyntactic/semantic specification, and N a vector in lexical space.*

A chain *is a sequence of LObjects* $\{L[1\ldots n],N[1\ldots n]\}$ *stored at LNodes* $P[1\ldots n]$*, such that* $N[i]=P[i+1]$*, and* $P[n+1]=_{df}\infty$.

For any vector **v**, $\mathbf{v}+\infty=_{df}\infty$.

In order to keep track of distributed LObjects in the lexicon we stipulate that they are stored as chains. Thus the lexical distribution implied by our analysis of the (verb) forms *ban* and *banned* should be

(3.20) ***Lexical distribution implied by* ((ban),d)**

1^ary^ *Key*	2^ary^ *Key*	*LNode*	*LObjects*
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	{[ban,v.],∞}⋆
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	{[ban,v.],($\mathbf{b}_1$,$\mathbf{d}_2$,$\mathbf{1}_3$)}⋆
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	{[past],∞}

This is how the table should be read: at LNode $(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)$ there is an LObject {[ban,v.],∞}⋆. This LObject is both the start of a (distributed LObject) chain (it is marked ⋆), and the end of a chain (it points to ∞). Therefore $(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)$ contains a trivial distributed LObject (consisting of only one LObject) whose meaning is [ban,v.]. At LNode $(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)$ there is an LObject {[ban,v.],$(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)$}⋆. This LObject too is the start of a chain. The next member is to be found at $(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)$; at $(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)$ we find the LObject {[past],∞}. This LObject is (correctly) not marked ⋆ since it is not the start of a chain. The LObject {[past],∞} points to ∞ and so is the last link in the chain. We have, then, a two member chain which encodes the distributed LObject ([ban,v.],[past]), which in turn we can calculate is syntactically equivalent to [ban,v.,past].

3.2.5 *Chains and the Barycentric curve*

Introducing chains introduces also the need for a more realistically defined notion of barycentric curve which takes into account the fact that mass is 'orientated' in lexical space (it points in definite directions). In fact, the necessary changes are quite straightforward, and simply extend the existing definitions, without contradicting them. The chief formal problem lies in the fact that chains may be of very different lengths.

We first recall that every PWord contains a primary key. Therefore every chain must contain at least one link which exists solely in primary space. We can use this common property as a baseline along which to align any chains we wish to compare, or any chains whose barycentre we wish to calculate. Where both chains have corresponding links, we can calculate their sum, or their barycentre, or whatever; where one chain lacks a corresponding link, we say that the chain is undefined at this point, and the sum/barycentric calculation proceeds as if the chain did not exist. Where chains exist at several points in primary hash space (like *kick the bucket*, for example), we need to ensure that each of the primary hashkey links is aligned. The acquisition of the barycentre then proceeds analogously to the example pursued at the end of the previous chapter: with each new acquisition, candidate phonological analyses are selected which minimise barycentric shift. The structure of the barycentre is, then, a curve of arbitrary length in the 36-dimensional Euclidean space which contains hashkey space.

For the rest of this Chapter we restrict our attention to chains which are encoded by a single PWord. We consider chains interpreted by more than one PWord in Chapter Four.

*   *   *

Because we know the computational properties of the hashtable, we have opened a considerable empirical window. If the distributed LObject hypothesis is correct, then we should expect to be able to predict and measure the resources used in their recovery. For example, the LNode lists that contain the distributed LObject that evaluates to [BAND,n.], should, if our analysis is correct, require a predictable quantity of computational resources more to recover than the LNode list containing the LObject [DOG,n.] (since in the first case a primary hashkey and two secondary hashkeys have to be accessed, whereas in the second case only one primary key and one secondary key have been accessed. The additional resources required are therefore equal to the resources required to perform a secondary hashkey look-up).

3.3 *Relatedness*

Another concern of the morphologist is the 'relatedness' of linguistic forms, and what constrains possible systems of relatedness. Given our commitment to *l'arbitraire du signe*, we prefer to pose the question differently and ask *why* things should be related? The answer would seem to be for efficient acquisition; that is, the efficient allocation of storage for new LObjects.

Thus we view morphology *qua* relatedness as a solution to the problem of memory allocation, specifically, the allocation of memory to calculated/hypothesised LObjects. We view this aspect of morphology, therefore, as a facet of language acquisition: from the hashtable location of an established LObject, another hashtable location needs to be found for new LObjects generated by the acquisition device from this LObject.

We discuss in this section some metatheoretical postulates which we suppose to be minimally necessary constraints on any theory which claims to model the relationships between morphosyntactic forms, between phonological forms and the inter-relationships between the two. We also instantiate these postulates by extending the theory we have been developing throughout this work.

We show in §3.4 following that the devices we have introduced, and which we suppose fulfil minimally necessary metatheoretical criteria, are acquirable in a biologically plausible way, and we provide an algorithm and an accompanying computational complexity analysis.

3.3.1 *Historical Perspective*

Much of the debate that dogs the progress of linguistics is rooted in the wildly divergent definitions of the objects over which linguistic relations are said to hold, and the nature of those relations. The same (erroneous) desire to 'conserve resources' which we noted in Chapter One seems to lie at heart of the persistence of some of the central dogma of generative phonology and morphology. We here take a brief but critical look at the usual lines of reasoning.

One of the main *credos* of generative accounts is that if *a* and *b* are related, then *a* and *b* are derived from some common source. In fact, 'derivation' (whether defined procedurally or declaratively) is the only relation defined over the set of well formed linguistic objects. We may then argue about the type of object that this common source is. Is it of the same type as *a* and *b*, or does it belong to some other type (or 'level of representation')? The usual hypothesis is that the common source is indeed at another level of representation, that is, it is of a different type to the related forms *a* and *b*. This assumption immediately gives rise to questions about this new level of representation—is it similar to the level at which *a* and *b* appear? If it is, why is it? These questions were discussed extensively in the post-SPE period, when notions of abstractness and recoverability were being explored (*e.g.* Kiparsky 1971, 1973, Kaye 1974). The distinction between underlying forms and surface forms is preserved to this day, and hence the seemingly endless debates about derivations and their constraints (see for example the papers collected in Roca 1997; Goldsmith 1993a; Durand & Katamba 1995:265–382).

The simplest hypothesis, however, is surely that the common source is of the same type as *a* and *b*. This idea has found more acceptance among morphologists than phonologists. For example Aronoff's pioneering work took it as axiomatic that

> All regular word-formation processes are word-based. A new word is formed by applying a regular rule to a single already existing word. Both the new word and the existing one are members of major lexical categories.
>
> — *Aronoff* 1976*:*21

The only phonologists to subscribe to the phonological analogue of this view are the Government Phonologists. In Government Phonology it is held that a phonological representation is independently interpretable at *any* point in a derivation (Harris & Lindsey 1995), Kaye's *Uniformity Condition:*

> Phonological representations are directly interpretable at every level.
>
> — *Kaye* 1995b*:*292

In other words, forms that are the 'input' to a derivational relation (the so-called *L-Structures*) are of the same type as the 'output' of that derivation (the *P-Structures*) (Kaye 1995b). In general, an L-structure in one language can be a perfectly good P-structure in another language, and *vice versa*.

However, this stance still preserves an artificial underlying/surface distinction for individual forms in individual languages which effectively assigns *two* phonological representations to a single word. Given that a phonological representation is a key in a lookup system, it seems bizarre that a single 'word' should have *two* keys, simply to perform a *single* lookup. Given also the discussions in Chapter Two, we take as our null hypothesis that there is *no* derivation from an 'underlying' to a 'surface' form, and hence that each 'word' has *exactly one* phonological representation, which is its hashkey.

We can still explore the idea that forms may be related. We take it that relatedness is simply a measure of similarity. In this way we do not need to posit common sources or derivations.

Postulate (3.21) Relatedness
Relatedness *is a measure of similarity (or distance) between two things.*

We have a ready-made metric defined over phonological forms, which is the phonological distance function introduced in Chapter Two. We discuss this further in §3.3.4–4 below, but first introduce some metatheoretical notions relevant to morphosyntactic relatedness.

3.3.2 *Modeling and Implementing Relatedness*

In Chapter One we introduced the basic unit of lexical information, the LObject. In Chapter Four we explore the nature of LObject content. For the purposes of this chapter, however, it is enough to know that an LObject contains a formal specification, giving information on the syntactic category of the LObject and any 'selection' features it may have (including, as we saw in Chapter Two, features for selection based on 'paralinguistic' properties such as 'informal', 'citation'), and a conceptual-semantic specification which we here leave undefined. We also introduce the auxiliary notion of *CSEquivalence*, such that two LObjects are CSEquivalent if they have the same conceptual-semantic structure.

Postulate (3.22) LObjects and CSEquivalence
An LObject contains a pair (C,F) where C is a conceptual-semantic structure, and F is a morphosyntactic structure.

LObjects (C,F) and (C,G), for any F,G are CSEquivalent.

We introduced earlier in this chapter the notion that LObjects exist in chains. It seems that we as native speakers are able to treat chains quite holistically (which we modelled by defining the syntactic equivalent of a chain, or its syntactic sum). Therefore we need some notion of relatedness between chains. What we introduce here is a notion of *translation* between chains, where intuitively two chains are translations if they have the same 'shape' and 'orientation' in space, but not necessarily the same physical location. We wish to capture, for example, the idea that *loved* and *waved* are translations. These forms are 'related', in that they are both past tenses formed with *-ed*.

The formulation of this idea comes in two parts: the LObject part and the PWord part. But first let us consider the distributed LObject encoded by the chain. The fact that certain LObjects appear in a distribution is itself a relationship between those LObjects. We propose here that there is a neat formulation of the idea that the relations between the members of a distributed LObject can be abstracted from the content that is specific to that particular distribution. This allows us the possibility of building new distributed LObjects analogously by superimposing the abstract distributed LObject structure onto specific content.

So what is the structure to be abstracted? Looking at both *loved* and *waved*, the common parts, and hence the abstractable parts, are precisely those parts of the distribution without conceptual-semantic structure. Let us therefore define the notion of a *virtual distributed LObject*, which is computable from any given distributed LObject **L** by substituting a special metavariable (which we call 'α') for the conceptual-semantic content of the distributed LObject **L**. In general we symbolise such an abstraction ($\mathbf{L}|\alpha$). So, from the distributed LObject $\mathbf{L}$=([BAN,v.],[past]) we can abstract the common distributed properties, getting ($\mathbf{L}|\alpha$)=([α,v.],[past]).

By replacing the metavariable α in ($\mathbf{L}|\alpha$) with conceptual-semantic content we can create new distributed LObjects which are related to **L**. Thus, taking the conceptual-semantic structure LOVE we can create the related distributed LObject ($\mathbf{L}|\alpha$=LOVE)=([LOVE,v.],[past]).

DEFINITION (3.23) PARADIGMS

For any distributed LObject **L** *with conceptual-semantic structre C, the* virtual counterpart to **L**, *written* ($\mathbf{L}|\alpha$)*, is identical to* **L** *except that every instance of 'C' in* **L** *is replaced by an instance of 'α' in* ($\mathbf{L}|\alpha$).

Another dimension along which we can measure LObject relatedness is 'paradigmatic'. That is, we would like to say that *love* and *loved* are related, because they are both 'forms' of *love*. We have also introduced the notion of a virtual distributed LObject, so we have as a consequence the notion of a *virtual paradigm*, which is precisely a paradigm whose members are all CSEquivalent to the metavariable α. Let us then introduce the notion of a (distributed) LObject paradigm, defined as an equivalence class over distributed LObjects:

Definition (3.24) Paradigms

A paradigm **P** with respect to conceptual-semantic structure *C is a set* $\mathbf{P}(C)=\{L \mid L^{+}$ *is CSEquivalent to* $C\}$, *for distributed LObjects L.*

A virtual paradigm *is a paradigm* $\mathbf{P}(\alpha)$, α *a metavariable over conceptual-semantic structures.*

Again, the virtual structure is useful as a means of generating new distributed LObjects, since if we have a paradigm {[ban,v.],([ban,v.],[past])}, from which we abstract the virtual paradigm $\mathbf{P}(\alpha)=\{[\alpha,\text{v.}],([\alpha,\text{v.}],[\text{past}])\}$, we can create things like $\mathbf{P}(\alpha=\text{LOVE})=\{[\text{LOVE},\text{v.}],([\text{LOVE},\text{v.}],[\text{past}])\}$.

The process of acquiring new distributed LObjects can therefore be modelled straightforwardly as follows. When a new distributed LObject *N* is encountered, we can identity its conceptual-semantic content *C*, create $(N|\alpha)$, and add $(N|\alpha)$ to the virtual paradigm. With *C* we can instantiate all the virtual distributed LObjects we have previously abstracted; and we can instantiate $(N|\alpha)$ with all the conceptual-semantic structures we have previously identified, thereby extending their paradigms analogously. We discuss this process in great detail in §3.4.

Now, as mentioned earlier, this is only half the story, since all these paradigms and virtual structures must be stored somewhere. The problem is interesting because of the relationship we have established between the phonology and the storage mechanisms, and because of the fact that the vast majority of LObjects are created (according to the algorithm above), rather than directly encountered. Thus storage has to be found automatically, which translates informally into the observation that the phonological forms of these newly created LObjects have to be computed. Until we can provide storage for an LObject, the rest of our cognitive faculties cannot interact with it. In particular, without being stored, it will not have a hashkey, and without a hashkey it will have no (phonetic) interpretation: we will have no way of knowing how to utter it. In §3.3.4 we explore how we come by the required phonological knowledge. What we need, in other words, is a phonological counterpart to virtual structure.

To implement a paradigm, we need to store the collection of morphosyntactically related distributed LObjects in a *linked list*. That is, a paradigm is stored in the lexicon as a list of chains. The order is not relevant, as any order should generate the same paradigm (there does not seem to be any difference between '*amo amas amat*' and '*amat amas amo*', *etc.*). The existence of such a paradigmatic pointer in a lexical specification is a 'promise' that going to the place pointed at will guarantee access to a morphosyntactically related form.

Unlike chains, it seems unnecessary to stipulate a given end point to a paradigm. Further, we have to ensure that no matter where we are in a paradigm, we can access *all* other members of the paradigm. We can do this simply by stipulating that the paradigm is not so much a linked list, but a *ring*. As long as we ensure that a single-member paradigm points to itself, we can append LNodes to the ring in the usual linked-list fashion, and maintain the ring property. We call these structures *LRings*.

So, we can redefine the content of an LNode more strictly as an *LObject record*, which we define as the structure $\{\mathbf{L},\mathbf{Q}\}$, for LObject **L** and LNode pointer (vector in lexical space) **Q**. Note the distinction between the two types of pointer (chain *vs.* LRing). They are independent of each other, and appear to serve very different purposes. The only aspect they do share is that they are both ontologically the same objects: pointers into lexical space. One type creates chains and is traversed 'syntagmatically'; the other type creates rings and is traversed 'paradigmatically'.

DEFINITION (3.25) LOBJECT RECORDS AND LRINGS

An LObject record *is a pair* $\{\{\mathbf{L},\mathbf{N}\},\mathbf{Q}\}$, *where* $\{\mathbf{L},\mathbf{N}\}$ *is an LObject and* **Q** *a vector in lexical space. The* **Q***-component is the* LRing-link.

An LRing *is a sequence of LObject records* $\{\mathbf{L}[1\ldots n]^\star,\mathbf{Q}[1\ldots n]\}$ *stored at LNodes* $\mathbf{R}[1\ldots n]$ *such that each LObject* $\mathbf{L}[i]^\star$ *is the start of a chain,* $\mathbf{Q}[i]=\mathbf{R}[i+1]$ *and* $\mathbf{R}[n+1]=\mathbf{R}[1]$, *for* $1\leq i\leq n$.

With this definition we can dispense with the metasymbol ⋆ for indicating the start of chain, since we now have that only the starting points of chains can appear on LRings, indicated by the presence of a non-∞ LRing pointer. So, assuming that *ban* and *banned* appear on the same LRing (they belong to the same paradigm) we can give a complete description of their lexical distribution as:

(3.26) ***Lexical distribution implied by* ((ban),d)**

1^{ary}	2^{ary}	*LNode*	*LObjects*
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN,V.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{BAN,V.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{\{[\text{past}],\infty\},\infty\}$

This table is read as follows: at LNode ($\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3$) is a one-member distributed LObject ([BAN,v.]). This distributed LObject is part of an LRing which points to the distributed LObject ([BAN,v.],[past]) stored in the chain beginning with the LObject record at ($\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3$). The LObject record {{[past],∞},∞} (the end of this chain) is not itself on the LRing (has LRing pointer ∞) because it is not the start of a chain.

3.3.3 *LRing Calculus*

With these stipulations in mind, we can now treat the general question of how to assign storage automatically to any distributed LObject. All distributed LObjects are stored as chains with links of the general form $\{L,N\}@M$, read 'LObject L pointing to the LNode N, and stored at LNode M'. To create a chain which is a translation of a given chain, we have to ensure that the orientations and mutual distances of each node are preserved. We do this by considering the quantity $N-M$, the directed distance from the current link to the next link. The virtual counterpart of $\{L,N\}@M$ is therefore $\{(L|\alpha),\beta+N-M\}@\beta$, for β a metavariable over LNode vectors. This ensures that in any instantiation of this virtual structure, the directed distances between the links of the chain so instantiated are identical with the directed distances between the links of the chain from which the abstraction was calculated.

In fact, we need to be a little more subtle, since we are dealing with a hashtable, whose entries are lists of things hashed to a given point in the table, and we are assuming that when new things are created, they are appended to existing lists. Hence, given an LNode vector N, let $H(N)$ be the hashtable position of N. The link in the virtual chain should therefore be more accurately defined as $\{(L|\alpha),(H(\beta)+H(N)-H(M))+\xi_3\}@\beta$, where ξ_3 is a metavariable over the third component of LNode vectors. We also note that $H(\infty)=_{df}\infty$. When we come to instantiate this virtual structure, the particular value of ξ_3 is given by some function which returns the next available free LNode on the list $(H(\beta)+H(N)-H(M),\#)$.

Now, we also need a counterpart to virtual LRings. Given that in an LRing, only one LObject record appears, we can treat the list of start-of-distributed LObjects just like we treated chains, and preserve the orientation and mutual distances between the links in the LRing. Thus, for an LRing link $\{\{L,N\},\mathbf{Q}\}@M$, we can calculate the virtual counterpart as $\{\{(L|\alpha),(H(\beta)+H(N)-H(M))+\xi_3\},\mathbf{Q}\}@\beta$. The value of $\mathbf{Q}$ is given by the operation which inserts LRing links into LRings.

DEFINITION (3.27) VIRTUAL LRING LINKS

For an LRing link $\{\{L,N\},\mathbf{Q}\}@M$, *the virtual counterpart is* $\{\{(L|\alpha),(H(\beta)+H(N)-H(M))+\xi_3\},\mathbf{Q}\}@\beta$.

DEFINITION (3.28) LRING INSERTION

Given an LRing link $\{\{L_1,N_1\},Q_1\}@M_1$ *in LRing* $\mathbf{L}$, *and given a new LRing link* $l_2=\{\{L_2,N_2\},\mathbf{Q}\}@M_2$ *to insert, the LRing resulting from inserting* l_2 *into* $\mathbf{L}$, $\mathbf{L}+l_2$, *contains links* $\{\{L_1,N_1\},M_2\}@M_1$ *and* $\{\{L_2,N_2\},Q_1\}@M_2$.

The lexicon of our simple example, incorporating virtual structure stored in the $\mathbf{0}_1$-hashtable, and where, simplifying, $(H(\beta)+H(x))+\xi_3=(\beta_1+x_1,\beta_2+x_2,\xi_3)$ looks like:

(3.29) ***Lexical distribution implied by*** **((ban),d)*, with virtual structure***

1^{ary}	2^{ary}	*LNode*	*LObjects*
β_1	β_2	ξ_3	$\{\{[\alpha,\text{v.}],\infty\},(\beta_1+\mathbf{0}_1,\beta_2+\mathbf{0}_2,\xi_3+\mathbf{1}_3)\}$
β_1	β_2	$\xi_3+\mathbf{1}_3$	$\{\{[\alpha,\text{v.}],(\beta_1+\mathbf{0}_1,\beta_2+\mathbf{d}_2,\xi_3)\},(\beta_1+\mathbf{0}_1,\beta_2+\mathbf{0}_2,\xi_3)\}$
β_1	$\beta_2+\mathbf{d}_2$	ξ_3	$\{\{[\text{past}],\infty\},\infty\}$
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{\{[\text{past}],\infty\},\infty\}$

The next step is to design an acquisitional algorithm which can create, exploit and manipulate this virtual structure to populate the lexicon with minimal supervision.

Since this notation is able to abstract any morphosyntactic relationship and generalise it over arbitrary chains we suppose that this characterisation of morphological knowledge is sufficient for an adequate theory. We turn in the next section to a thorough investigation of how large networks of chains and LRings can be acquired semi-automatically from learning stimuli that we may suppose real human beings are exposed to.

3.4 *Acquisition*

In this section we give a worked example of the early stages of acquisition of LE, which shows how new LObjects are created, assigned storage, and occasionally abandoned. We follow closely the format adopted in Chapter Two, and use the lexical distributions already established earlier in this chapter. In this way we build up a theory of acquisition, which culminates in a completely general acquisition algorithm. We hope that concentrating on the small snippet of LE here shows that each aspect of the algorithm we propose is a necessary one.

3.4.1 *Attested Mass*

Attested mass is the mass (*q.v.* Chapter Two) we learn directly from the environment. Assume first that we have successfully acquired and analysed from the environment the following (trivial) distributed LObjects: {[BAN,V.],∞}, {[PEE,V.],∞}. We could have done this encountering the forms ban *ban* and piː *pee* in some suitably explicit context. Each LObject so attested we stipulate has a mass of 1. We need to allocate these objects storage in the lexicon as LObject records. The hashtable location is given by the phonological forms, which we let be ($\mathbf{b}_1$,$\mathbf{0}_2$) and ($\mathbf{p}_1$,$\mathbf{0}_2$) respectively.

We suppose that there must be a mechanism which inserts distributed LObjects into the lexicon. This operation needs to find currently unused LNodes at which the members of the distributed LObjects are to be stored, ensure that the pointers are consistent, and incorporate the resulting chain into the appropriate LRing.

The phonological form of a distributed LObject gives us the hashed lists of LNodes (($\mathbf{b}_1$,$\mathbf{0}_2$,**#**) and ($\mathbf{p}_1$,$\mathbf{0}_2$,**#**)) into which to insert the new members of the chain. Assume then that there is a function *next*(*x*) which evaluates to the (position vector) of the next available LNode on the list hashed to *x*. So, beginning at the start of each chain we are to insert, we insert the LObject records {{[BAN,V.],∞},**Q**}@($\boldsymbol{x}_1$,$\boldsymbol{x}_2$,$\boldsymbol{x}_3$), where ($\boldsymbol{x}_1$,$\boldsymbol{x}_2$,$\boldsymbol{x}_3$)=*next*(($\mathbf{b}_1$,$\mathbf{0}_2$)), and {{[BAN,V.],∞},**R**}@($\boldsymbol{y}_1$,$\boldsymbol{y}_2$,$\boldsymbol{y}_3$), where ($\boldsymbol{y}_1$,$\boldsymbol{y}_2$,$\boldsymbol{y}_3$)=*next*(($\mathbf{p}_1$,$\mathbf{0}_2$)) for as yet unknown LRing pointers **Q**and **R**.

The two LObjects we have encountered do not share the same conceptual-semantic structure, hence do not belong to the same paradigm, so both new LObject records can start life as separate (length one) LRings. Therefore, because both LObject records are the starts of chains, we can introduce the LObject records {{[BAN,V.],∞},($\boldsymbol{x}_1$,$\boldsymbol{x}_2$,$\boldsymbol{x}_3$)}@($\boldsymbol{x}_1$,$\boldsymbol{x}_2$,$\boldsymbol{x}_3$)), where ($\boldsymbol{x}_1$,$\boldsymbol{x}_2$,$\boldsymbol{x}_3$)=*next*(($\mathbf{b}_1$,$\mathbf{0}_2$)) and {{[BAN,V.],∞},($\boldsymbol{x}_1$,$\boldsymbol{x}_2$,$\boldsymbol{x}_3$)}@($\boldsymbol{y}_1$,$\boldsymbol{y}_2$,$\boldsymbol{y}_3$)), where ($\boldsymbol{y}_1$,$\boldsymbol{y}_2$,$\boldsymbol{y}_3$)=*next*(($\mathbf{p}_1$,$\mathbf{0}_2$)). We therefore have the following lexical distribution:

(3.30) ***Lexical distribution Ia***

1^{ary} *Key*	2^{ary} *Key*	*LNodes*	*Content*	*Mass*
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	{{[BAN,V.],∞},($\mathbf{b}_1$,$\mathbf{0}_2$,$\mathbf{1}_3$)}	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	{{[PEE,V.],∞},($\mathbf{p}_1$,$\mathbf{0}_2$,$\mathbf{1}_3$)}	1

Similarly, if we acquire another LObject meaning for *pee*, say (the name of the letter 'P'), the storage allocation mechanism would append it to the LNode list $(\mathbf{p}_1,\mathbf{0}_2,\#)$, given by $\{\{[\text{'P'},\text{n.}],\infty\},(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)\}@(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)$, where $(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)=next((\mathbf{p}_1,\mathbf{0}_2))$:

(3.31) ***Lexical distribution Ib***

1*ary* *Key*	2*ary* *Key*	*LNodes*	*Content*	*Mass*
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3),(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{PEE},\text{v.}],(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{'P'},\text{n.}],(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1

But before we go any further, we need to explore how virtual structure can be created. Let us therefore rewind history a little and imagine we have only acquired the single form *ban*, with the lexical distribution:

(3.32) ***Initial Lexical distribution***

1*ary* *Key*	2*ary* *Key*	*LNodes*	*Content*	*Mass*
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1

Our definition of virtual structure gives us immediately the form $\{\{[\alpha,\text{v.}],\infty\},\mathbf{Q}\}@(\beta_1,\beta_2,\beta_3)$, where $(\beta_1,\beta_2,\beta_3)=next((\beta_1,\beta_2))$. Let us also assume that there is a virtual counterpart to mass, which does not enter into barycentric calculations, for which we use the symbol μ_0. There is no prior virtual structure, so the form is a new virtual LRing, and therefore needs to point to itself. Our lexicon now looks like:

(3.33) ***Initial Lexical distribution with virtual mass***

1*ary* *Key*	2*ary* *Key*	*LNodes*	*Content*	*Mass*
β_1	β_2	β_3	$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1

We now have a virtual structure from which we can potentially instantiate (non-virtual) structure. We must be careful of a number of subtleties when instantiating LObjects from virtual structure: we may create chains for which there is no allocatable storage (the hashtable vector sums take them outside the hashtable); if an LObject so created is allocatable, this allocation is achieved through insertion into linked lists of various sorts (LNode lists, chains, LRings). In other words, the third component of all LNode pointers is *not* determined by virtual structure, but by the insertion operation, which must find empty LNodes using the *next* function.

In order to instantiate a virtual LObject record we need two things: a conceptual-semantic specification (to instantiate 'α'}, and a hashtable location (to instantiate '(β_1,β_2)'). This is precisely the sort of information that we assumed in Chapter One would be provided by a *thesaurus*. In this case the required thesaurus is accessed through conceptual-semantic structures. Let us then assume the following crude description of the conceptual-semantic thesaurus (the *CSThesaurus*): each listed conceptual-semantic structure σ accesses a record which contains a pointer to a hashtable location. This hashtable location in turn contains a list of LNodes, on which we are guaranteed to find an LObject record with the corresponding conceptual-semantic content σ. By the definition of LRings, we will automatically have access to all the morphosyntactically related forms of σ simply by traversing the LRing. Let us assume then the process of acquisition also involves updating the CSThesaurus, and at the stage we are at now we have:

(3.34) ***Initial CSThesaurus***

CS-structure	*Hashtable Location*
BAN	$(\mathbf{b}_1,\mathbf{0}_2)$

Now that we know where to find the required information we can examine how it is used to instantiate virtual structure. In a virtual structure $\{\{(L|\alpha),N\},\mathbf{Q}\}@(\beta_1,\beta_2,\beta_3)$, the conceptual-semantic structure is used to instantiate α, and the hashtable location is used to instantiate β_1 and β_2. $\mathbf{Q}$ is given entirely by the operation of LRing insertion, as usual.

In general, there are two types of instantiation, often encountered simultaneously. When a new (previously unknown) piece of virtual structure is abstracted, this new virtual structure can be used to generate analogous forms from the existing (non-new) repertoire of conceptual-semantic structures in the CSThesaurus. Secondly, when a new CSThesaurus entry is attested (rather than created), it can be used to instantiate the entire virtual structure repertoire (insofar as the instantiated structures have well-defined hashtable positions). This we believe is the heart of the acquisition process, as it allows the rapid population of the lexicon by generating and storing related objects.

Also, when creating virtual structure from an attested form there are in general two strategies. Firstly, if the CSThesaurus already contains the conceptual-semantic structure of the newly attested form, then by definition a paradigm, and hence an LRing already exists into which the new form must be inserted. If it is not in the CSThesaurus, then the new form should be inserted as a brand new LRing. The strategy used directly affects the values of the LRing pointers, and hence must *precede* the creation of virtual structure.

Now assume that we encounter *P* (the name of the letter: $\{\{[\text{'P'},\text{n.}],\infty\},\mathbf{Q}\}@(\mathbf{p}_1,\mathbf{0}_2,\#)$). Before we do anything else we need to see whether it can be inserted into an existing LRing, or whether a new LRing needs to be created to accommodate it. By checking the CSThesaurus we note that the expected entry 'P':$(\mathbf{p}_1,\mathbf{0}_1)$ does not exist. We therefore insert it into the CSThesaurus. The result so far is as follows, where we adopt the convention, in displaying intermediate distributions, of marking brand new records with the metasymbol †.

(3.35) ***Intermediate CSThesaurus* (I)**

	CS-structure	*Hashtable Location*
	BAN	$(\mathbf{b}_1,\mathbf{0}_2)$
†	'P'	$(\mathbf{p}_1,\mathbf{0}_2)$

Next, we can perform an instantiation, since we have a new CSThesaurus record, and we have non-new virtual structure. Instantiating the virtual structure gives us $\{\{[\text{'P'},\text{v.}],\infty\},\mathbf{Q}\}@(\mathbf{p}_1,\mathbf{0}_2,\boldsymbol{x}_3)$, where $(\mathbf{p}_1,\mathbf{0}_2,\boldsymbol{x}_3)=next((\mathbf{p}_1,\mathbf{0}_2))$. Given the existing distribution, we have that $\boldsymbol{x}_3=\mathbf{1}_3$. We leave it to the semantics, or some other higher authority to decide whether or not ['P',v.] is a useful or 'meaningful' LObject. We simply note that such an LObject has been created, and it is quite possible to imagine a scenario that it could be used: *We shall mark all residents' car parking spaces with the letter 'P'; if a space hasn't been P-ed, you can't park in it.*

There is no existing LRing into which we can incorporate this instantiation so we can directly insert the LObject record into the lexicon as a brand new single-member LRing $\{\{[\text{'P'},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\boldsymbol{x}_3)\}@(\mathbf{p}_1,\mathbf{0}_2,\boldsymbol{x}_3)$, where $(\mathbf{p}_1,\mathbf{0}_2,\boldsymbol{x}_3)=next((\mathbf{p}_1,\mathbf{0}_2))$.

(3.36) ***Intermediate lexical distribution* (I)**

1^{ary} *Key*	2^{ary} *Key*	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{'P'},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1

Next we consider in its entirety the LObject we encountered, $\{\{[\text{'P'},\text{v.}],\infty\},\mathbf{Q}\}@(\mathbf{p}_1,\mathbf{0}_2,\boldsymbol{x}_3)$, where $(\mathbf{p}_1,\mathbf{0}_2,\boldsymbol{x}_3)=next((\mathbf{p}_1,\mathbf{0}_2))$. From the table above we have $\boldsymbol{x}_3=\mathbf{2}_3$. By definition this new object exists on an LRing (its conceptual-semantic content is in the CSThesaurus), so we need to insert it into the LRing pointed to by the CSThesaurus entry ‘P’:$(\mathbf{p}_1,\mathbf{0}_1)$. Using the operation of LRing insertion given previously, we manipulate the LRing pointers accordingly, getting:

(3.37) ***Intermediate lexical distribution* (II)**

1^{ary} *Key*	2^{ary} *Key*	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{‘P’},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	†	$\{\{[\text{‘P’},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1

Next we can create the virtual counterpart of the newly attested LObject record (which we have just created). From the definition of abstraction we have $\{\{[\alpha,\text{n.}],\infty\},\mathbf{Q}\}@(\beta_1+\mathbf{0}_1,\beta_2+\mathbf{0}_2,\beta_3)$, where, as usual, $(\beta_1,\beta_2,\beta_3)=next((\beta_1,\beta_2))$. We are careful to note that every addition of virtual structure is by definition an insertion, so we perform the appropriate pointer manipulation, and achieve the following:

(3.38) ***Intermediate Lexical distribution* (III)**

1^{ary} *Key*	2^{ary} *Key*	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$	†	$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{‘P’},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	†	$\{\{[\text{‘P’},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1

Finally, we can complete this acquisitional cycle by using all the non-new conceptual-semantic structure in the CSThesaurus to instantiate this new virtual structure. Again, by definition, each newly created object will have an LRing into which it can be inserted. The by now familiar process gives us the instantiated LObject $\{\{[\text{BAN},\text{v.}],\infty\},\mathbf{Q}\}@(\mathbf{b}_1,\mathbf{0}_2,\boldsymbol{x}_3)$. Inserting this record completes one acquisitional cycle, and we can dispense with the daggers. The final distributions are:

(3.39) ***Final Lexical distribution***

1^{ary} *Key*	2^{ary} *Key*	*LNodes*	*Content*	*Mass*
β_1	β_2	β_3	$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$	$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{BAN},\text{n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{‘P’},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{‘P’},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1

(3.40) ***Final CSThesaurus***

CS-structure	*Hashtable Location*
BAN	$(\mathbf{b}_1,\mathbf{0}_2)$
'P'	$(\mathbf{p}_1,\mathbf{0}_2)$

So far we have considered a very simple example. There is a little more work involved in two areas: one is in the acquisition of chains longer than length one; another is the instantiation of paradigms longer than one LObject record. It turns out that no extra machinery is involved, but a little effort is required in the pointer 'book-keeping'.

Now, assume that a form band *banned* is encountered, and is successfully parsed and analysed (aided, perhaps by felicitous environmental circumstances) into the chain-encoded distributed LObject $(\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\boldsymbol{y}_3)\},\mathbf{Q}\}@(\mathbf{b}_1,\mathbf{0}_2,\boldsymbol{x}_3),\{[\text{past}],\infty\},\infty\}@(\mathbf{b}_1,\mathbf{d}_2,\boldsymbol{y}_3))$, where $\boldsymbol{x}_3$ and $\boldsymbol{y}_3$ are given from *next*$((\mathbf{b}_1,\mathbf{0}_2))$ and *next*$((\mathbf{b}_1,\mathbf{d}_2))$ respectively, and $\mathbf{Q}$ is given by the LRing insertion operation (recall that only the start of a chain contains a non-∞ LRing pointer). Checking the lexicon reveals that we have not encountered this particular distributed LObject before, so it is a new form. So far all is much as we have seen above.

Now, a distributed LObject by definition contains only a single conceptual-semantic structure, which in this case is BAN. According to the CSThesaurus, this form already exists, so the chain we have is going to have to be inserted into an existing LRing; in particular we are to insert it after the LObject record pointed to by the CSThesaurus entry BAN:$(\mathbf{b}_1,\mathbf{0}_2)$. So, manipulating pointers according to the usual method gives us $\mathbf{Q}$, and inserting the entire chain into the lexicon gives us values for $\boldsymbol{x}_3$ and $\boldsymbol{y}_3$. The intermediate lexical distribution is therefore

(3.41) ***Intermediate Lexical distribution***

1^{ary} *Key*	2^{ary} *Key*	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$		$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{BAN},\text{n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	†	$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	†	$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{'P'},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{'P'},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1

Now we need to create the virtual structure corresponding to the newly attested chain, and then instantiate it with all the other conceptual-semantic structures we know. To create a virtual chain, we simply begin with the start of the chain we wish to abstract, and create the virtual counterpart to each of the chain links in turn. The virtual structure is easily calculated from our previous definitions as $(\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\xi_3)\},\mathbf{Q}\}@(\beta_1,\beta_2,\zeta_3),\{\{[\alpha,\text{v.}],\infty\},\infty\}@(\beta_1,\beta_2+\mathbf{d}_2,\xi_3))$ which we can add to the virtual LRing in the usual fashion, getting

(3.42) ***Intermediate Lexical distribution***

1^{ary}	2^{ary}	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{2}_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$		$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{2}_3$	†	$\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\beta_3)\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_0
β_1	$\beta_2+\mathbf{d}_2$	β_3	†	$\{\{[\text{past}],\infty\},\infty\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{BAN},\text{n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	†	$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	†	$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{'P'},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{'P'},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1

The last act of this acquisitional cycle is to instantiate this new virtual structure with all the other conceptual-semantic structure we have in the CSThesaurus. Again the instantiation is straightforward, and we have two new LObject records to insert into the lexicon, into the LRing containing 'P'-words:

(3.43) ***Final Lexical distribution***

1^{ary}	2^{ary}	*LNodes*	*Content*	*Mass*
β_1	β_2	β_3	$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{2}_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$	$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{2}_3$	$\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\beta_3)\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_0
β_1	$\beta_2+\mathbf{d}_2$	β_3	$\{\{[\text{past}],\infty\},\infty\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{BAN},\text{n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{'P'},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{'P'},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{'P'},\text{v.}],(\mathbf{p}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	1

The owner of this lexicon now has a past tense for the verb *to 'P'*. The phonetic interpretation of the PWord which encodes the corresponding chain $((\mathbf{p}_1)\mathbf{d}_2)$, is ˈpʰɪːd.

Now assume we learn the form siː, *see*, whose chain we work out to be $\{\{[\text{SEE},\text{v.}],\infty\},\mathbf{Q}\}@(\mathbf{s}_1,\mathbf{0}_2,\boldsymbol{x}_3)$. This time we discover that there is no SEE-entry in the CSThesaurus, so we first of all have to instantiate existing virtual structure, using a new CSThesaurus entry SEE:$(\mathbf{s}_1,\mathbf{0}_2)$. Sparing the reader the by now familiar details, we get the following intermediate distribution:

(3.44) ***Final Lexical distribution***

1*ary*	2*ary*	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{2}_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$		$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{2}_3$		$\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\beta_3)\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_0
β_1	$\beta_2+\mathbf{d}_2$	β_3		$\{\{[\text{past}],\infty\},\infty\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{BAN},\text{n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$		$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$		$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{'P'},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{'P'},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$		$\{\{[\text{'P'},\text{v.}],(\mathbf{p}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$		$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{SEE},\text{v.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	†	$\{\{[\text{SEE},\text{n.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	†	$\{\{[\text{SEE},\text{v.}],(\mathbf{s}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	†	$\{[\text{past}],\infty\},\infty\}$	1

When we come to insert the actual attested chain, we find that the instantiation has already created it for us. That is, the attested form does not generate any new virtual structure. This particular is cycle is therefore at an end, and the above distribution is also the final distribution.

At this point the owner of this lexicon is happy to say that the past tense of *see* is *seed* (we also have a noun *see* well, which the author at least can conjure up in a phrase like *let me have a see*). But how is it that mature speakers come to have *saw* as the generally accepted past tense?

3.4.2 *Distributed Mass*

All is well, until the time that we encounter the form ˈsɔː *saw*, and discover that it means [SEE,v.,past]. The chain that the acquisition device has to store is $\{\{[\text{SEE},\text{v.},\text{past}],\infty\},\mathbf{Q}\}@(\mathbf{a}_1,\mathbf{0}_2,\boldsymbol{x}_3)$. Stepping through the algorithm just as we did in the previous subsection to the point where we use the new virtual structure to generate more chains from the existing conceptual-semantic structure in the CSThesaurus gives us:

(3.45) ***Intermediate Lexical distribution***

1*ary*	2*ary*	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1+\mathbf{a}_1-\mathbf{s}_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$		$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{2}_3$		$\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\beta_3)\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_0
β_1	$\beta_2+\mathbf{d}_2$	β_3		$\{\{[\text{past}],\infty\},\infty\}$	μ_0
$\beta_1+\mathbf{a}_1-\mathbf{s}_1$	β_2	β_3	†	$\{\{[\alpha,\text{v.},\text{past}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{2}_3)\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{BAN},\text{n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$		$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$		$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{'P'},\text{v.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{'P'},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$		$\{\{[\text{'P'},\text{v.}],(\mathbf{p}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$		$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{SEE},\text{v.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{SEE},\text{n.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$		$\{\{[\text{SEE},\text{v.}],(\mathbf{s}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$		$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{a}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{SEE},\text{v.},\text{past}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1

Now, instantiating the new virtual chain with 'P':$((\mathbf{p}_1,\mathbf{0}_2))$ gives us the following potential past tense for *to 'P'*: $\{\{[\text{'P'},\text{v.},\text{past}],\infty\},\mathbf{Q}\}@(\mathbf{p}_1+\mathbf{a}_1-\mathbf{s}_1,\mathbf{0}_2,\boldsymbol{x}_3)$. Now, if we adopt the phonetic interpretation function for LE given in Chapter Two (§2.8.2), we have the following more specific phonological definitions of $\mathbf{b}_1$, $\mathbf{p}_1$, $\mathbf{s}_1$ and $\mathbf{a}_1$.

(3.46) ***Phonological forms***

Vector symbol	*Value*	*Interpretation*
$\mathbf{s}_1$	$(\mathbf{ACD},\mathbf{0},\mathbf{BCG},\mathbf{0})$	ˈsɪː
$\mathbf{a}_1$	$(\mathbf{ACD},\mathbf{0},\mathbf{BCEF},\mathbf{0})$	ˈsoː
$\mathbf{b}_1$	$(\mathbf{ADF},\mathbf{0},\mathbf{BD},\mathbf{0})$	ˈba^n
$\mathbf{p}_1$	$(\mathbf{CF},\mathbf{0},\mathbf{BCG},\mathbf{0})$	ˈp^{h}ɪː
$\mathbf{p}_1+\mathbf{a}_1-\mathbf{s}_1$	$(\mathbf{CF},\mathbf{0},\mathbf{BCEF},\mathbf{0})$	ˈp^{h}oː
$\mathbf{b}_1+\mathbf{a}_1-\mathbf{s}_1$	$(\mathbf{ADF},\mathbf{0},\mathbf{BDEF}-\mathbf{G},\mathbf{0})$	???

Note that the vector defined by $\mathbf{b}_1+\mathbf{a}_1-\mathbf{s}_1$ lies outside the hashtable, and so there is no way to allocate space for the instantiation of the new virtual structure using BAN:($\mathbf{b}_1,\mathbf{0}_2$). Hence the LRing containing BAN-forms does not get augmented in this acquisitional cycle. The final distribution is therefore as follows, where daggers have been retained for the reader's convenience.

(3.47) ***Final Lexical distribution***

1^{ary}	2^{ary}	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1+\mathbf{a}_1-\mathbf{s}_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$		$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{2}_3$		$\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\beta_3)\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_0
β_1	$\beta_2+\mathbf{d}_2$	β_3		$\{\{[\text{past}],\infty\},\infty\}$	μ_0
$\beta_1+\mathbf{a}_1-\mathbf{s}_1$	β_2	β_3	†	$\{\{[\alpha,\text{v.,past}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{2}_3)\}$	μ_0
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{BAN,v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{BAN,n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$		$\{\{[\text{BAN,v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$		$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$		$\{\{[\text{'P',v.}],\infty\},(\mathbf{p}_1+\mathbf{a}_1-\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$		$\{\{[\text{'P',n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$		$\{\{[\text{'P',v.}],(\mathbf{p}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$		$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1+\mathbf{a}_1-\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{'P',v.,past}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{SEE,v.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	†	$\{\{[\text{SEE,n.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	†	$\{\{[\text{SEE,v.}],(\mathbf{s}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	†	$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{a}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{SEE,v.,past}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1

Now we are in an interesting position, since there are now two distributed LObjects in some of the LRings which have exactly the same syntactic equivalent. For example we have *saw* and *seed* for [SEE,v.past] and '*paw*' and *'P'-ed* for ['P',v.past]. Note that we view it in general as a good thing that the 'strange' (with hindsight) forms like *seed* and *paw* are generated because with only a little effort (usually that associated with doing a crossword puzzle, or designing a linguistic experiment) the past tense meanings are recoverable by most speakers, and the analogy on which they are made seems in general to be available to speakers. In fact there is evidence that 'incorrect' forms often gain some common currency: *brung* (for *brought*), and jocular *thunk* for *thought*.

Assume, then, that we entertain both the choices (we provide a choice mechanism later) and say that they are in *competition*. We need to introduce some mechanism which can arrive at a choice between competitors, based on evidential experience.

Definition (3.48) Competing Chains

Chains L and M are competitors *if and only if* $L^+ \equiv M^+$

Let us assume, then, that one competitor is preferred over another if its mass is greater. That is, we can use mass as a measure of 'strength' of a competitor.

However, competitors are strictly speaking chains, which can be of varying lengths, and hence of varying total masses. But we do not wish to say that some competitor is intrinsically better than another simply because it is longer. Intuitively we want all competitors to start out equal. A simple solution is to exploit the fact that the links in an LRing consist of exactly one LObject record (the start of a chain). Let us therefore agree to use the starts-of-chains as 'representatives' as it were of the entire chain, and adopt the convention that whatever happens to the mass of the start of the chain LObject record, the *same* mass adjustment happens to all members of the chain.

Since competitors are in some sense fighting for the same bit of space in the paradigm, we model this by saying that they are all fighting over the same, fixed, quantity of mass. The start-of-chain LObject records of competing members of an LRing we say form a *mass distribution*, with the property that the total mass of all the members of the distribution equals 1.

Definition (3.49) Mass Distributions

Let n LObject records l_i *be the starts of competing chains* L_i *and let* $m(l_i)$ *be the mass of* l_i*, then the* mass distribution $m(l_1) + \ldots m(l_n) = 1$.

For every non-start member x of a chain L with start member l,
$m(x)=m(l)$.

Adding a form to a mass distribution should preserve the existing mass ratios of the existing members of the mass distribution (after all, they reflect all the experience acquired to date). Call the mass of the new form (the competing LObject to be introduced into the LRing) x. The rest of the masses in the existing distribution have to give up x amount of mass, in order to conserve the total distributional mass when the new form comes into the distribution. Assume each mass gives up the same proportion of its own mass as exists between it and the total distribution mass. That is, each mass $m[i]$ gives up $xm[i]$. So the revised masses are $m'[i]=m[i](1-x)$.

The question is, what is the mass x? If it is too big, it will be too disruptive of the established distribution, and if it is too small it may never stand a chance of competing properly. We therefore stipulate that the new mass is the mean mass of the new distribution. That is, for an existing distribution of n LObject records, we would add another LObject record with a mass of $(n+1)^{-1}$. The other masses are then adjusted to $m'[i]=m[i](1-(n+1)^{-1})$. Any such mass adjustments are then transmitted down the chain headed by the i LObject records in the mass distribution.

Definition (3.50) Mass Insertion
Let n LObject records l_i be the starts of competing chains L_i and let
$\boldsymbol{M}=m(l_1)+\ldots m(l_n)=1$. Given chain C whose start is c and $m(c)=\boldsymbol{\mu}$,
we define the mass distribution $\boldsymbol{M}+C=m'(l_1)+\ldots m'(l_n)+m'(c)=1$,
where $m'(l_i)=m(l_i)(1-\boldsymbol{\mu}(n+1)^{-1})$, and $m'(c)=\boldsymbol{\mu}(n+1)^{-1}$.

We can therefore give an updated display of the contents of our lexicon, by adjusting the masses of the competing chains (*seed/saw*, *'P'-ed/'paw'*). Note that we cannot give virtual masses like $\frac{1}{2}\boldsymbol{\mu}_0$ since for some given CSThesaurus entry it may be impossible to instantiate all competing members of the virtual LRing (as was the case with BAN:($\mathbf{b}_1$,$\mathbf{0}_2$)). We adopt the convention, therefore, of using subscripts to indicate forms belonging to the same mass distribution, and leave it to each particular instantiation to allocate mass evenly such that the distribution sums to 1, according to the definitions given above.

(3.51) ***Final Lexical distribution with revised masses***

1^{ary}	2^{ary}	*LNodes*	*Content*	*Mass*
β_1	β_2	β_3	$\{\{[\alpha,\text{v.}],\infty\},(\beta_1+\mathbf{a}_1-\mathbf{s}_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$	$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_1
β_1	β_2	$\beta_3+\mathbf{2}_3$	$\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\beta_3)\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	μ_2
β_1	$\beta_2+\mathbf{d}_2$	β_3	$\{\{[\text{past}],\infty\},\infty\}$	μ_2
$\beta_1+\mathbf{a}_1-\mathbf{s}_1$	β_2	β_3	$\{\{[\alpha,\text{v.,past}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{2}_3)\}$	μ_2
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{BAN},\text{n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{'P'},\text{v.}],\infty\},(\mathbf{p}_1+\mathbf{a}_1-\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{'P'},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{'P'},\text{v.}],(\mathbf{p}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	½
$\mathbf{p}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	½
$\mathbf{p}_1+\mathbf{a}_1-\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{'P'},\text{v.,past}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	½
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{SEE},\text{v.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{SEE},\text{n.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{SEE},\text{v.}],(\mathbf{s}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	½
$\mathbf{s}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	½
$\mathbf{a}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{SEE},\text{v.,past}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	½

In the absence of any further direct evidence we might assume that that is how things remain, and a genuine arbitrary choice between *seed* and *saw* exists.

However, we do not need to assume that further direct evidence is unavailable. What happens then if some other form, *loved*, is encountered? One thing will be the 'rediscovery' of the *ban/banned* paradigm. We can assume that this reapplication 'reinforces' the *-ed* paradigm, biasing our choice (but perhaps only briefly) in favour of *-ed* past tenses. We look in detail at the further evolution of the acquisitional process, and introduce a mechanism which models 'reinforcement' by adjusting the masses in mass distributions.

3.4.3 *Adjusting Mass*

Of course, as acquisition progresses, we certainly become able to decide whether or not to accept *'P'-ed* as the past tense of *'P'*. In the language of our theory that means that the mass of *'P'-ed* must increase somehow at the expense of '*paw*'. We assume further that these mass adjustments are made in the light of evidential experience, and that the conservation of mass property of the distribution is preserved (3.49).

Assume, then, that certain events trigger a reinforcement of one LObject record, and the corresponding atrophy of the competing form(s). We therefore need to increase the mass of the reinforced competitor at the expense of the others, while conserving total mass across all competitors. Say there are n competing LObject records in a distribution and assume we should increase the mass of the reinforced candidate by amount α. We accordingly need to deduct this amount of mass across the other candidates.

Now, consider the mass distribution $m[1]+\ldots m[n]=1$, of these n LObject records. Without loss of generality, we let $m[n]$ be the mass to be reinforced. We need to share between the losers the amount α of mass to be surrendered to LObject record n. Assume each loser gives up an amount of mass proportional to its size relative to the other losers (a smaller loser yields a smaller amount of mass than a more massive loser). The scaled mass distribution of the losers is given from the total mass of the losers $m[1]+\ldots m[n-1]=1-m[n]$, hence $(m[1]+\ldots m[n-1])(1-m[n])^{-1}=1$. Let $\beta=(1-m[n])^{-1}$. From each loser we need to subtract the appropriate proportion of α. Hence for $(0<i<n)$, set $m'[i]=m[i](1-\alpha\beta)$, and set $m'[n]=m[n]+\alpha$, for the new mass distribution m'.

Note that we should have $0\leq m[i](1-\alpha\beta)\leq 1$ and $0\leq(m[n]+\alpha)\leq 1$. This means some careful selection of a function for α. We can achieve this if we ensure that the increase due to α is some fraction of the total mass of the distribution *not* assigned to $m[n]$. That is, $\alpha=\gamma^{-1}(1-m[n])$, for $\gamma\geq 1$. Since $\alpha\beta=\gamma^{-1}(1-m[n])(1-m[n])^{-1}$, this simplifies the readjustment rule to $m'[i]=m[i](1-\gamma^{-1})$. The choice of γ seems to be axiomatic. We choose the fraction of the unused mass which the winner takes from the losers to be equal to the ratio of the winner's mass to the total mass of the distribution. Hence we choose γ^{-1} to be equal to $m[n]$. Thus we have the reinforcement law $m'[n]=m[n]+m[n](1-m[n])$, hence $m'[n]=m[n](2-m[n])$; and the atrophy law $m'[i]=m[i](1-m[n])$.

Definition (3.52) Reinforcement and Atrophy

For a mass distribution $m[1]+\ldots m[n]=1$, *we define* reinforcement *and* atrophy *to give the new distribution* $m'[1]+\ldots m'[n]=1$ *as follows:*

Reinforcement: $m'[n]=m[n](2-m[n])$;

Atrophy: $m'[i]=m[i](1-m[n])$.

We use the following convenient notation to indicate multiple reinforcements: for a mass m, $M^p(m)$ indicates the result of p applications of the reinforcement law on m; $W^p(m,n)$ indicates the result of atrophying the mass m p times, given mass n is correspondingly reinforced p times.[1]

Note that a mass of 1 cannot be reinforced further, and if it is for some reason atrophied, it becomes 0 immediately. A mass of 0 cannot be reinforced. As we would expect, this seems to be well interpreted by removing any LObject records with this mass from the lexicon. A mass of 0 means you do not exist.

What is the event that triggers the adjustment of mass? We assume that each encounter of a form which already has mass reinforces that form. Thus in our example, every favourable encounter of the form *saw* reinforces the LObject record $\{\{[\text{SEE},\text{v.},\text{past}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}@(\mathbf{a}_1,\mathbf{0}_2,\mathbf{1}_3)$, at the expense of its competitors. However, each encounter with *saw* should not reinforce the counterpart past tense in other paradigms, since it introduces no new paradigmatic information (virtual structure).

However, every time we encounter a form that has not previously been encountered, we create a virtual counterpart to it, as we have seen. If this virtual counterpart does in fact already exist in virtual structure, we can apply the reinforcement and atrophy laws to the virtual mass distribution in which it occurs. In this way we can reinforce particular paradigmatic links by encountering new instances of them. We also stipulate that if any virtual mass is adjusted, there is some mechanism which can propagate the mass adjustment 'retrospectively' to existing forms in the lexicon which instantiate the reinforced virtual structure, and which atrophy any of its competitors accordingly.

One simple way to do this is at the appropriate intermediate stages of an acquisitional cycle to mark the reinforced and atrophied virtual structure with corresponding metasymbols (we choose ⋆ and ‡ respectively), and have a final subcycle which uses the CSThesaurus entries and virtual structure to locate all the instantiations of the adjusted virtual masses, and reinforces or atrophies those masses accordingly.

To illustrate, assume we encounter the form ˈwiːd *weed*, which we assume is the chain $(\{\{[\text{WEE},\text{v.}],(\mathbf{w}_1,\mathbf{d}_2,\mathbf{1}_3)\},\mathbf{Q}\}@(\mathbf{w}_1,\mathbf{0}_2,\boldsymbol{x}_3), \{\{[\text{past}],\infty\},\infty\}@(\mathbf{w}_1,\mathbf{d}_2,\boldsymbol{x}_3))$. We first note that the CSThesaurus does not contain an entry for WEE, so we need to instantiate existing virtual structure with WEE:$(\mathbf{w}_1,\mathbf{d}_2)$. We note in passing that the quantity $\mathbf{w}_1+\mathbf{a}_1-\mathbf{s}_1$ is in the hashtable (and is pronounced ˈwɔː *waw*). When we instantiate the competitive virtual mass distribution (the μ_2 distribution) we are careful to respect the definition of a mass distribution by distributing the allocatable mass of 1 evenly across all competitors. The new forms we have are

(3.53) ***Partial intermediate lexical distribution***

1^ary	2^ary	*LNodes*		*Content*	*Mass*
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{WEE},\text{v.}],\infty\},(\mathbf{w}_1+\mathbf{a}_1-\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	†	$\{\{[\text{WEE},\text{n.}],\infty\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	†	$\{\{[\text{WEE},\text{v.}],(\mathbf{w}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	½
$\mathbf{w}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	†	$\{[\text{past}],\infty\},\infty\}$	½
$\mathbf{w}_1+\mathbf{a}_1-\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{WEE},\text{v.},\text{past}],\infty\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	½

According to our algorithm we now try to insert the entire newly encountered form into the lexicon. However, we find that an appropriate chain already exists (it has just been created as an instantiation of virtual structure). However, our mass adjusting algorithm states that every (external) encounter with an existing form causes that form to be reinforced. So we reinforce the masses in the *weed*-chain, at the expense of its competitor *waw*. From our definitions we have that M(½)=¾ and W(½,½)=¼. We therefore have

(3.54) ***Partial intermediate lexical distribution with adjusted masses***

1^ary	2^ary	*LNodes*		*Content*	*Mass*
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{WEE},\text{v.}],\infty\},(\mathbf{w}_1+\mathbf{a}_1-\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	†	$\{\{[\text{WEE},\text{n.}],\infty\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	†	$\{\{[\text{WEE},\text{v.}],(\mathbf{w}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	¾
$\mathbf{w}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	†	$\{[\text{past}],\infty\},\infty\}$	¾
$\mathbf{w}_1+\mathbf{a}_1-\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	†	$\{\{[\text{WEE},\text{v.},\text{past}],\infty\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	¼

When we come next to incorporating the virtual counterpart into the virtual LRing we notice that it already exists (the virtual counterpart is $(\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\zeta_3)\},\mathbf{Q}\}@(\beta_1,\beta_2+\mathbf{d}_2,\xi_3),\{\{[\text{past}],\infty\},\infty\}@(\beta_1,\beta_2+\mathbf{d}_2,\zeta_3)))$. Since this is our first encounter with WEE-words we are required to adjust the masses in the competitive virtual mass distribution μ_2, and mark them with the appropriate metasymbols:

(3.55) ***Partial intermediate virtual distribution with adjusted masses***

1^ary	2^ary	*LNodes*		*Content*	*Mass*
β_1	β_2	β_3		$\{\{[\alpha,\text{v.}],\infty\},(\beta_1+\mathbf{a}_1-\mathbf{s}_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$		$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_1
β_1	β_2	$\beta_3+\mathbf{2}_3$	⋆	$\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\beta_3)\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	$\mathrm{M}(\mu_2)$
β_1	$\beta_2+\mathbf{d}_2$	β_3	⋆	$\{\{[\text{past}],\infty\},\infty\}$	$\mathrm{M}(\mu_2)$
$\beta_1+\mathbf{a}_1-\mathbf{s}_1$	β_2	β_3	‡	$\{\{[\alpha,\text{v.},\text{past}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{2}_3)\}$	$\mathrm{W}(\mu_2)$

Taking all the non-new entries in the CSThesaurus, we next instantiate the flagged virtual structures and locate the instantiations in the lexicon. We then atrophy or reinforce these entries according to the metasymbol. This completes the current acquisitional cycle, and we have the final distribution:

(3.56) ***Final CSThesaurus***

CS-structure	*Hashtable Location*
BAN	$(\mathbf{b}_1,\mathbf{0}_2)$
'P'	$(\mathbf{p}_1,\mathbf{0}_2)$
WEE	$(\mathbf{w}_1,\mathbf{0}_2)$

(3.57) ***Final virtual distribution***

1*ary*	2*ary*	*LNodes*	*Content*	*Mass*
β_1	β_2	β_3	$\{\{[\alpha,\text{v.}],\infty\},(\beta_1+\mathbf{a}_1-\mathbf{s}_1,\beta_2,\beta_3)\}$	μ_0
β_1	β_2	$\beta_3+\mathbf{1}_3$	$\{\{[\alpha,\text{n.}],\infty\},(\beta_1,\beta_2,\beta_3)\}$	μ_1
β_1	β_2	$\beta_3+\mathbf{2}_3$	$\{\{[\alpha,\text{v.}],(\beta_1,\beta_2+\mathbf{d}_2,\beta_3)\},(\beta_1,\beta_2,\beta_3+\mathbf{1}_3)\}$	$\text{M}(\mu_2)$
β_1	$\beta_2+\mathbf{d}_2$	β_3	$\{\{[\text{past}],\infty\},\infty\}$	$\text{M}(\mu_2)$
$\beta_1+\mathbf{a}_1-\mathbf{s}_1$	β_2	β_3	$\{\{[\alpha,\text{v.,past}],\infty\},(\beta_1,\beta_2,\beta_3+\mathbf{2}_3)\}$	$\text{W}(\mu_2)$

(3.58) ***Final lexical distribution***

1*ary*	2*ary*	*LNodes*	*Content*	*Mass*
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{BAN},\text{v.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{BAN},\text{n.}],\infty\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{BAN},\text{v.}],(\mathbf{b}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{b}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	1
$\mathbf{b}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{`P'},\text{v.}],\infty\},(\mathbf{p}_1+\mathbf{a}_1-\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{`P'},\text{n.}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{p}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{`P'},\text{v.}],(\mathbf{p}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	¾
$\mathbf{p}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	¾
$\mathbf{p}_1+\mathbf{a}_1-\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{`P'},\text{v.,past}],\infty\},(\mathbf{p}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	¼
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{SEE},\text{v.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{SEE},\text{n.}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{SEE},\text{v.}],(\mathbf{s}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	¾
$\mathbf{s}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	¾
$\mathbf{a}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{SEE},\text{v.,past}],\infty\},(\mathbf{s}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	¼
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{WEE},\text{v.}],\infty\},(\mathbf{w}_1+\mathbf{a}_1-\mathbf{s}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{2}_3$	$\{\{[\text{WEE},\text{n.}],\infty\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{1}_3)\}$	1
$\mathbf{w}_1$	$\mathbf{0}_2$	$\mathbf{3}_3$	$\{\{[\text{WEE},\text{v.}],(\mathbf{w}_1,\mathbf{d}_2,\mathbf{1}_3)\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{2}_3)\}$	¾
$\mathbf{w}_1$	$\mathbf{d}_2$	$\mathbf{1}_3$	$\{[\text{past}],\infty\},\infty\}$	¾
$\mathbf{w}_1+\mathbf{a}_1-\mathbf{s}_1$	$\mathbf{0}_2$	$\mathbf{1}_3$	$\{\{[\text{WEE},\text{v.,past}],\infty\},(\mathbf{w}_1,\mathbf{0}_2,\mathbf{3}_3)\}$	¼

Notice that our device now prefers *seed* over *saw*, even though it directly attested *saw* earlier on. It has effectively 'regularised' the past tense for the time being. It would take another two direct attestations of *saw* to bring its mass over ½, (since $M^2(¼)=0.68359375$) and hence to be preferred over *seed* again. The explicitness of our theory allows us to make claims like this, and in fact claims at a far coarser resolution, some of which we look at briefly in §3.5. Before that, however, we summarise and formalise the acquisitional algorithm.

3.4.4 *The acquisitional algorithm*

The illustrative and cumulative approach to developing a theory of acquisition we have been taking thus far can quickly become unwieldy, and can give the impression of using subtle deductions to take computational short-cuts. We therefore use this subsection to tidy things up a little by spelling out the various stages of an acquisitional cycle in detail. From this rigorous specification of the acquisitional algorithm we will be able to make some general claims about acquisitional behaviour, and the complexity of the acquisition process, and make some interesting predictions. These reflections are the subject of §3.5 following.

A cycle is initiated by encountering a PWord which we are able to parse into a corresponding chain, *L*. Next we check to see if *L* is already in the lexicon. If it is, we simply reinforce the masses in the existing chain, and atrophy their competitors; the cycle ends.

If it is not already in the lexicon, we check to see if its conceptual-semantic structure is in the CSThesaurus. If it is not, we enter it into the CSThesaurus (flagged with † as being new); we then instantiate all existing virtual structure with this new entry and insert it all into the lexicon as a brand new LRing (instantiation and insertion require care over maintaining the conservative property of mass distributions).

Next, we attempt to insert *L* into an LRing in the lexicon; if *L* already exists because we have just instantiated it, we reinforce its mass, and atrophy the masses of its competitors; otherwise we insert it into the appropriate LRing.

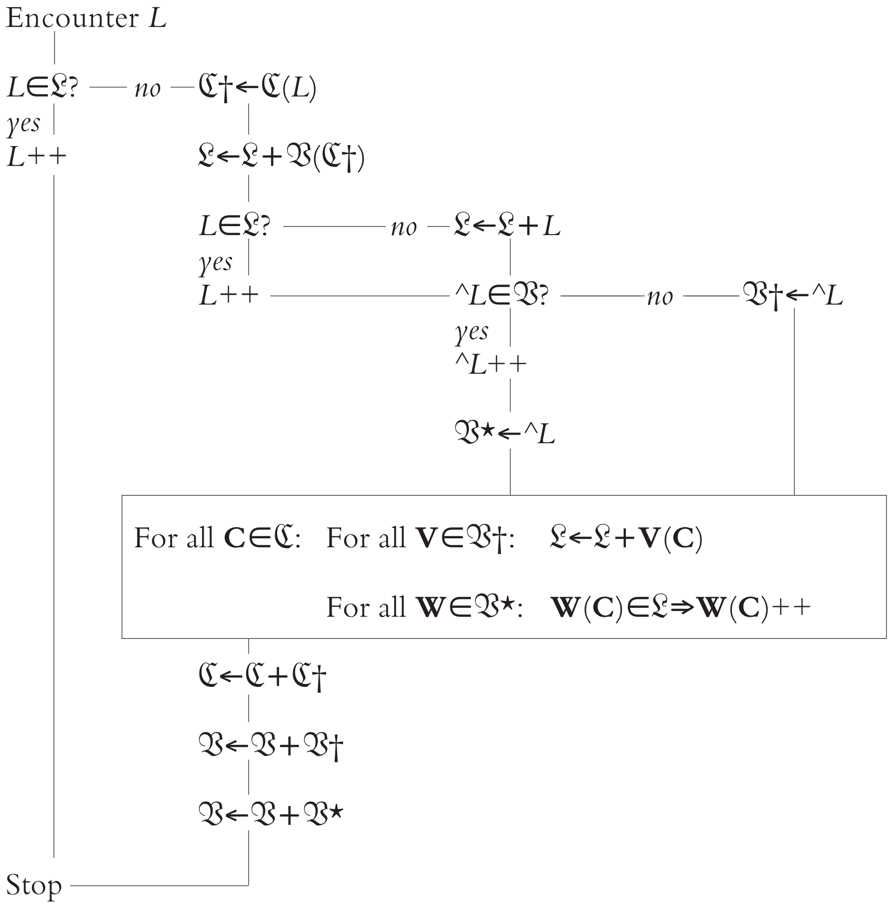
Next, we abstract the virtual counterpart of *L*, which we call ^*L*. If ^*L* already exists in virtual structure we reinforce its virtual mass and flag it with ⋆, and atrophy the virtual masses of its competitors; otherwise we insert it into the virtual LRing, flagged † (insertion requires care over maintaining the subscripts indicating virtual mass distributions).

Next, we take each non-† entry in the CSThesaurus, instantiate the †-marked virtual structure and insert it into the lexicon; then we instantiate the ⋆-marked virtual structure, locate it in the lexicon, and reinforce its mass and atrophy the masses of its competitors.

Finally, all metasymbol instances († and ⋆) are deleted; the cycle ends.

The acquisitional process can now be shown diagrammatically with the following additional notational aids: $\mathfrak{V}(\mathbf{C})$ is the set of all well-defined chains generated by instantiating existing virtual structure $\mathfrak{V}$ with CSThesaurus entry **C**. Similarly $\hat{}L(\mathbf{C})$ is the result of instantiating a virtual structure $\hat{}L$ with **C**; $\mathfrak{C}$ is all existing entries in the CSThesaurus, and $\mathfrak{L}$ is the current content of the lexicon. The notation $X \leftarrow X+Y$ is to be read 'insert *X* into *Y*', and *X*++ is to be read 'reinforce *X* and atrophy its competitors'. $\mathfrak{C}(L)$ is the conceptual semantic structure of *L* (*i.e.* $L \equiv \hat{}L(\mathfrak{C}(L))$). $X^\star$ is ⋆-flagged structure and $X^\dagger$ is †-flagged structure.

(3.59) ***Acquisitional cycle***



The computational properties of this algorithm are perfectly acceptable. The most complex terms are of the order of the total size of the existing lexicon, which potentially increases with every application of the algorithm. However, the increase is only ever at most equal to the sum of the size of virtual structure (a new form is encountered, and a paradigm generated for it) and the size of the CSThesaurus (a new paradigmatic link is spread throughout the lexicon). It follows from the definition of virtual structure that the size of the CSThesaurus is always some fraction of the size of virtual structure. So at any given time t in the history of the device (where one unit of acquisitional time is defined by an encounter with an acquirable datum), let the size of the lexicon be $m(t)$, the size of virtual structure is $v(t)$; then the size of the CSThesaurus is no greater than $v(t)$. The size of the lexicon at $t+1$ therefore satisfies $m(t+1) \leq m(t)+2v(t+1)$.

Note that according to the algorithm for an encountered form L consisting of n LObjects, virtual structure can increase by at most n in one action of the algorithm. Assume, not unreasonably, that in the situation of a human acquirer, no L containing more than N LObjects is considered as a suitable input. We have $v(t) \leq N+N+\ldots N$, t times, hence $v(t) \leq Nt$, hence $m(t+1) \leq m(t)+Nt+N$. Further, initial $m(0)=0$ (we have no pre-existing forms when we begin), so we can solve the recurrence to give $m(t) \leq \frac{1}{2}N(t^2+3t-2)$. Hence the computational complexity of the acquisition of morphological knowledge is $O(t^2)$, assuming that virtual structure can grow without limit.

We note from our knowledge of the world, however, that as time increases, the likelihood that v increases decreases (we are less and less likely to encounter new paradigmatic forms as our experience lengthens). So there is some limit to which v tends as t increases. Call this limit $|\mathfrak{V}|$. We therefore have that $m(t+1) \leq m(t)+|\mathfrak{V}|$. Solving the recurrence gives $m(t) \leq t|\mathfrak{V}|$. Hence, the computational complexity actually tends to $O(t)$ as t increases. We can therefore declare that the computational complexity of our model of the acquisition of morphological knowledge decreases towards $O(t)$ with increasing t.

This result accords well with observation: just after the time when even a rudimentary ability to parse a PWord and identify chains has been acquired, we witness ‘explosion’, where computational resources (particularly space) are quickly consumed with hypothesised forms. But the rate of this activity decreases as less and less new information becomes available from the environment, until ‘observable’ acquisition all but ceases. We should ultimately be able to relate these results directly to the observation of the allocation of neuronal resources in the human brain during acquisition, a topic for future research.

3.5 *Macro-Morphology*

At the macro-level, our theory allows us to analyse the evolutionary trends of acquisition algorithms. One interesting question is Why is there not more 'irregular' morphology? Our theory allows for systems which abound in irregular morphology; yet a survey of the world's languages would seem to indicate that superficially at least, related LObjects tend to be stored apparently fairly systematically under closely related hashkeys.

As time goes by, the more often a new instance of a particular virtual paradigmatic link is encountered, the more that link will be reinforced throughout the lexicon. In order to preserve an existing competing form in the lexicon that is not an instance of this link (an 'irregular' form, say), we should have to ensure that we encounter many reinforcing instances of that particular form. This seems to be precisely what is observed in empirical studies of language acquisition. It is noted that 'irregular' forms typically have a high 'token frequency' (many occurrences of the same form), whereas 'regular' patterns have a high 'type frequency' (many occurrences of novel instances of the virtual structure).

According to our theory, as the structure matures, virtual mass acts as a record of type frequency, thus new forms will be entered into the lexicon instantiating an LRing with the accumulated biasing of previous encounters with particular types of paradigmatic link. Once in the lexicon, the actual mass of a form reflects the bias of accumulated encounters with exactly that form.

Thus we may generalise that 'irregular' morphology can be sustained, *ceteris paribus*, in high frequency forms. We do *not* have to say that there is in general any pressure to 'regularise'; regularisation would only tend to happen to very low (token) frequency forms.

Further, as the device's experience broadens, fewer and fewer new forms instantiating existing virtual paradigmatic links will be encountered, so that over the course of maturation, reinforcements due to type frequency should lessen, giving way in importance to reinforcements due to token frequency. This actually implies that it should be possible to observe 'over-regularisation' at the early stages of acquisition as 'regular' forms are reinforced with each encounter of a new instance of an existing virtual paradigmatic link, only giving way to the reinforcement of 'irregular' forms later on. Empirical evidence, such as it is, would seem to agree with this. This also explains neatly how it is possible to have 'irregular' forms with apparently low token frequency (like *weave/wove*): these forms *must* be acquired late in the acquisition process, when the reinforcements due to type-frequency are significantly scarcer (in English this is certainly the case. Most of us first learn *wove* when we are practically at school age!).

Note also that the high frequency of an existing token has no effect on the frequency of the types it instantiates.

Of course, given enough time, it would be quite possible to produce/acquire a language with more irregular than regular morphology. Although history has provided us with some plausible candidates (*e.g.* Old Irish, especially the verbal system, Thurneyson 1949), our theory predicts that the situations where this is likely to occur must be very special indeed, namely a prolonged period of interaction with the linguistic environment that must extend well beyond the usual two or three years that gives us the predominance of 'regular' morphology we see in the world's languages.

Thus, our theory provides us with an interesting empirical opportunity, as it directly relates intensity/length of the period of acquisition with the (likelihood of) ir-/regularity of the resulting morphology. Further predictions include that on the average, the shorter/less intensive the period of acquisition, the more 'regular' the resulting language is likely to be (a study of the acquisition of Creoles in this framework should prove fruitful).

Further, if the investment of acquisitional effort in acquiring a language like Old Irish is too expensive (in some well-defined sense), we might expect that by cutting the investment (by shortening the learning period, for example), the language would mature before it had a chance to acquire all the 'irregularities'. Result, Old Irish > Middle Irish > Modern Irish, with their greatly simplified ('regular') verbal systems.

Of course what the stimulus to cut short the acquisitional period is is an interesting question, but not, I suspect, one that can be answered by linguists. Historians, sociologists and zoologists are probably better placed to understand the external pressures on organisms that determine the particular way they allocate resources to ensure their survival in a particular environment.

⋆ ⋆ ⋆

Our proposal, although computationally simple, does imply that in the maturation of a human language, a substantial amount of monitoring of the environment is called for. This is apparently counter to the long held belief in generative linguistics that this interaction is in fact minimal, and degraded ('Plato's problem, then, is to explain how we know so much, given that the evidence available to us is so sparse' Chomsky 1986:*xxvii*). However, we should note the following: there is considerable empirical basis for the claim that there is prolonged linguistic interaction during maturation (Gallaway & Richards 1994); the type of interaction our theory requires is a very 'passive' one. We don't need to hypothesise that it is only forms addressed *to* the child that are used; as long as forms can be identified anywhere in the environment in general, this usage can be used to adjust the weights of hypothesised forms.

Further, by our hypothesis, the acquisition device is able to generate most of the forms it will ever need all by itself, especially in the early stages of the process (§3.4.5) (which accords well with the spirit of Plato's problem). Our device, however, over-generates. We view the role of prolonged interaction as one of adjusting the masses of these forms, to see which ones are best suited to our environment ('the ones our parents like'). Again, the phenomenon of over-generalising is observed in children; and over-generation and subsequent atrophy of a proportion of the initial population is a tried-and-tested strategy found in natural mechanisms (Holland 1992).

We should recall, as we did in Chapter One, that the human brain is an immensely powerful computer, and that it works on the solution of Plato's problem for nearly two years, if not more.

Finally, it is worth underlining the fact that the formal definition of our theory is completely independent of the language, LE, we used to illustrate (and indeed motivate) it. The algorithm is completely general, and therefore, we claim, able to acquire the lexicon (*qua* morphological knowledge) of *any* language from the appropriate linguistic environment.

The building of a suitable computer implementation of the algorithm discussed in this chapter should not be a difficult task, and its construction and the construction of suitably encoded databases of multi-language acquisitional stimuli is a crucial next step in testing the bold universalist claim we make. We maintain that the proposals of this chapter represent a sound and innovative basis for launching such a programme.

3.5 *Notes to Chapter Three*

1. The equations in (3.52) are well-known in the mathematics of nonlinear and chaotic dynamical systems, where they define the so-called *logistcic model*. The basic shape of M^p for increasing p, for the values of m that interest us, is an S-curve between 0 and 1.

Such models have been found useful in the analysis of competitve behaviour in fields as diverse as economics (*e.g.* Brock & Malliaris 1989) and neural-net design (*e.g.* Aleksander & Morton 1990).

Chapter Four

Syntax

kurizaqadanaaĝiituudahlilakaq
'I do not even want to try to quit (habitual) smoking'
— *Bergsland* 1994

In this chapter we draw together all the threads of the preceding chapters and develop a theory of how syntactic information is packaged into phonological representations.

Up to now we have seen that our minimal approach to the architecture of human language does not require hypothesising a phonology which transformationally manipulates phonological representations. In fact we have seen that postulating just such a mechanism leads to insurmountable logical difficulties. It would be surprising therefore to find phonological rules needed in the domain of connected speech. However, much effort has been expended by linguists on the study of the phonology of words when they are strung together in utterances, and the general consensus seems to be that there are phonological rules which transform the 'dictionary form' of a word into the appropriate connected-speech-form. Here we reassure ourselves that proposing phonological rules of this type is the retrograde step that the logic tells us it is.[1]

No new formal apparatus needs to be introduced in the phonology. However, there are a number of important points where traditional and current assumptions about the nature of syntactic processing come under question. We make no attempt to develop fully a syntactic theory in the 'computational approach' used for the phonology and morphology. We do however outline some of the *desiderata* of such a theory and introduce enough of a formal metatheory to provide a foundation for more in-depth work:

Lexicon: no structure is abstracted, all information is available at lookup; therefore there is no need to have a syntax which 'fills in' structure, or derives necessary structure from underspecified inputs. Syntax only has to combine partial trees.

Syntax: a calculus for representing and assembling complexes of conceptual-semantic information. There is no derivative relationship between syntax and phonology. Phonology is the access mechanism to the lexicon.

Syntactic objects (hence ultimately conceptual-semantic complexes) are utterable because their assembled parts are stored in the lexicon, and have a phonological key. Insofar as there are alternative wellformed assemblages, there are genuinely alternative locutions (*I am happy*, *I'm happy*).

Variation: The primitives (partial trees) of languages may vary, but the atoms from which they are made do not. Primitives are particular configurations of atoms. Primitives are accessed via PWords, thus languages may vary as to whether they access an LObject through a bound form or a free form. There are apparently arbitrary controls on assembly (morphosyntactic categories; agreement; subcategorisation), which appear to be orthogonal to conceptual-semantic structure.

Acquisition: language specific decompositions; language specific control features. They are the 'morphology' of Chapter Three, and hence are generalisable, in answer to Plato's problem.

We discuss each of these aspects in the following sections.

4.1 *Prerequisites for a Syntactic Theory*

An exhaustive theory of the relationship between LObjects and their phonological forms ultimately requires being able to control, precisely, all aspects of both the phonological *and* the syntactic sub-theories. In this respect we are in agreement with Pollard and Sag, who argue eloquently for the necessity of explicitness in the scientific study of language (Pollard & Sag 1994:1–14). Unfortunately, existing off-the-shelf syntactic theories are, for a variety of reasons, unsuitable.

An area that is particularly important to a study such as ours, which makes certain claims about the computational architecture of the human language faculty, is the structure of the lexicon. We saw in earlier chapters that metatheoretical decisions of quite fundamental and far-reaching importance can be taken as a direct result of one's idea of how big the lexicon is. Our own investigations led us to reject the prevalent 'paucity of resources' hypothesis outright. This leaves us in something of a quandary when it comes to existing syntactic theories. Insofar as a 'redundancy free' lexicon is accepted as one of the axioms of a theory, the union of that theory and our theory will obviously be a contradiction. We have no choice, then, but to build our own syntactic theory. We shall of course find inspiration in existing theories, as well as some innovations of our own, necessitated by the radical change in perspective we are advocating. We find this approach far more wholesome than taking an existing theory, and trying to excise the 'small lexicon' axiom and all its consequences, and all those facets of the theory which in some way can be attributed to it, before bolting what's left of it on to ours.

For example, one candidate syntactic theory is the HPSG of Pollard and Sag (Pollard & Sag 1994). In certain obvious superficial ways it is inappropriate for our study: it considers phonological representations to be an integral part of a lexical item, thereby making them potentially available to the same calculus that manipulates the content of lexical items. In Chapter Two we saw that this assumption leads to considerable difficulties. However, the formalism of HPSG is not specific to linguistics (see, for example, Shieber 1992), and a version of it without phonological representations could be easily devised. More seriously, however, is that the version of HPSG presented by Pollard and Sag is as mistaken about the nature of the lexicon as, say, is the Minimalist approach (*cf.* Chapter One). It is clear that they believe that

> … properties of lexical entries and relationships among them are expressed in a concise and principled fashion in terms of classification by a multiple inheritance hierarchy … and lexical (redundancy) rules… respectively.
>
> — *Pollard & Sag* 1994:36

Our investigations in Chapters Two and Three of the present work have shown, however, that not only is the construction of 'rules' fraught with difficulties, but that systematic and quasi-systematic relationships between LObjects can be understood as the by-products of the acquisitional process of populating the lexicon. Given that all linguistic theories, including HPSG, need a theory of acquisition, we can justly claim that the mechanisms proposed for the HPSG lexicon carry a considerable *onus probandi*. Further, insofar as the supposed nature of lexical entries influences the particular choices of feature structures and unification strategies made by theoreticians, we cannot be sure that by adopting, say, Pollard and Sag's HPSG that we are not introducing inconsistencies into our own theory. So, while the HPSG formalism is flexible enough to be bent to our will, we underline here the importance of *not* adopting existing syntactic theories (however successful or attractive we may find them) without careful analysis of their logical compatibility with the basic assumptions of our own work.

4.1.1 *The lexicon*

The lexicon was studied in detail in Chapter One, but we remind ourselves here of the impact the results have on the design of syntactic theories. No structure is abstracted, all information is available at lookup; therefore there is no need to have a syntax which 'fills in' structure, or derives necessary structure from underspecified inputs. Syntax only has to combine partial trees.

4.1.2 *Syntactic metatheory*

It is possible to make some interesting generalisations, and even predictions, without developing a full-scale syntactic theory, but rather concentrating on the metatheory. That is, in order to be consistent with the findings of the previous chapters, what must a syntactic theory look like?

Any syntactic theory provides a set of atoms and an operation that combines them in some way. Let $\mathbf{S}=\langle A,\oplus\rangle$ be such a theory. Let us define a syntactic structure to be an *assemblage*, which we can define recursively as follows:

DEFINITION 4.1. ASSEMBLAGES

For any syntactic theory $\mathbf{S}=\langle A,\oplus\rangle$*, the set* $A^\star$ *of syntactic structures, or* assemblages *satisfies*

i. if a is in A then a is in $A^\star$*.*

ii. if a and b are in $A^\star$ *then* $a\oplus b$ *and* $b\oplus a$ *are in* $A^\star$*.*

In the physical world, we interpret syntactic atoms partly by conceptual-semantic structures, and thus we can view **S** as a calculus for assembling complexes of conceptual-semantic information.

From our discussion about the lexicon we know that **S** does not have to supply underspecified or abstracted information. The content of an LObject is, then, a fully specified syntactic structure. We might ask whether an LObject is simply a syntactic atom, or whether it is an assemblage. This question is, in fact, the key to understanding the phonology of connected speech. Our null hypothesis, given *l'arbitraire du signe*, must be that there is no restriction on what an LObject can 'mean', and hence that LObjects contain assemblages.

We can support this hypothesis with a simple demonstration that the alternative hypothesis is false. Assume that LObjects can only contain syntactic atoms. It follows that for any sequence of LObjects containing atoms *a* and *b*, such that *a* and *b* combine, giving $c=a\oplus b$, there is no (single) LObject containing *c*. Assuming, for example, that the LE PWords ˈdɪd *did* and ˈsiː *see* access LObjects which contain corresponding atoms, then there should be no single-lookup PWord which accesses the LObject containing *did*⊕*see*. Other things being equal, this would appear to be false, given the

LE PWord ˈsoː *saw*. The locution *I did see* is readily paraphrased *I saw*, and *vice versa*.

We are therefore confident that our null hypothesis is more in the nature of a corollary of linguistic arbitrariness, and so assert it as such:

Corollary 4.3. LObject Content
An LObject contains an assemblage.

In Chapters One and Two we learnt that there is no derivative relationship between syntax and phonology. Phonology is the access mechanism to the lexicon. Syntactic assemblages (hence ultimately conceptual-semantic complexes) are utterable because their assembled parts are stored in the lexicon, and there is a phonetic interpretation function which knows how to utter the hash keys which access the lexicon.

Therefore, in order to utter an assemblage, we have to be able to *decompose* it into a combination of assemblages which exist in the lexicon as LObjects. Insofar as there are alternative decompositions, there are genuinely alternative locutions (*I am happy*, *I'm happy*). Equivalently we have that an utterance is a sequence of combinable assemblages; these assemblages are accessed in the lexicon through PWords; hence an utterance can be realised as the phonetic interpretation of a sequence of PWords.

Recalling the properties of phonetic interpretation of Chapter Two, we should expect, then, that for some individuals (with the appropriate ASR-envelopes), at the point where the phonetic interpretation of one PWord ends and the following begins, we may be able to observe (the restricted class of) automatic/subconscious contact phenomena, like nasal place assimilation (see further Chapter Two).

4.2 *Variation*

It follows directly that a significant locus of syntactic variation is precisely in the content of LObjects. This is because there are in general many decompositions of a (complex) assemblage, and an LObject contains a particular component assemblage. Thus the decompositions that are available to an individual are precisely those for whose component assemblages there exist LObjects in the individual's lexicon.

Our theory indicates that this variation must manifest itself in the following way. Let language L_1 have LObjects containing a and b, in PWords **a** and **b**; let language L_2 have an LObject containing $a \oplus b$, in PWord **c**; and let language L_3 have LObjects containing a,b and $a \oplus b$ in PWords **d**,**e** and **f**.

Given an assemblage $a \oplus b$, language L_1 realises it with the two-PWord utterance [**a**][**b**], while language L_2 realises it through the one-PWord utterance [**c**], and language L_3 has two alternative realisations, as a 'syntactic' complex [**d**][**e**], or as a single 'word' [**f**]. We can say that **ab**, **c**, **de** and **f** are *translations* of each other, and 'mean' $a \oplus b$.

Another locus of variation given by our theory is whether an LObject is accessed through a bound or a free form. Thus we should expect languages L_4 having LObjects containing *a* and *b*, in PWord [[**g**]**h**]; languages L_5 having LObjects containing *a* and *b*, in PWord [**i**[**j**]]; and languages with combinations of all the above.

Hence our theory leads us to expect to find languages which realise the same syntactic structure in a variety of ways, from a single 'unanalysable' form (like [**c**]), through a 'morphological complex' (like [[**g**]**h**]) to a 'syntactic complex' (like [**a**][**b**]). This would appear to be corroborated by observation.

Now we can analyse the pervasive belief that there are special rules for connected speech phonology. If a language possesses locutions [**a**][**b**] and [[**a**]**h**] for the same assemblage $a \oplus b$, they will have been acquired in accordance with the theory of Chapter Three. In such a case virtual structure would contain $(b|\alpha)@(\beta_1,\beta_2+\mathbf{h},\#)$, hence the 'suffix' **h** 'meaning *b*' would be propagated throughout the lexicon. Insofar as a believer in phonological rules can create a rule that maps **b** into **h**, they will be able to claim that **h** is some 'reduced' or 'sandhi'-form of **b**. For our theory, however, there is no difference in kind between these forms and the more traditional 'morphological' forms studied in Chapter Three. In fact, the discussions of Chapter Three were undertaken with no specific syntactic or morphological theory in mind, and hence are completely general. All these forms simply form part of a LRing, and are not even to be considered a 'special case'.

We conclude, then, that the existence of apparent connected speech phenomena follows directly from the hypotheses that the lexicon is a (large) $\mathfrak{H}(3)$-hashtable, that phonology is the lexical hashing-interface, that phonology is interpretable, that syntax is compositional, and *l'arbitraire du signe*.

An objection that can be raised by proponents of such rules is that the resulting LRings of our theory are quite large, and that because 'sandhi'-forms appear in the lexicon a single phonological form may be much more ambiguous than in a rule-based theory, meaning a different order of computational effort is required to parse an utterance. The first objection is easily dispensed with, since we showed in Chapter One that all the evidence points to there being no significant limit on lexical storage space, and we have provided a computationally well-motivated model of storage, access and population of the lexicon.

The second objection is surprisingly not true. We prove this in the following section.

4.3 *Ambiguity*

Ambiguity resolution poses a computational problem for all theories. Let us imagine that in parsing a string of n objects, each of the n objects is on average p-ways ambiguous. The original (ambiguous) string is therefore potentially p^n unambiguous strings. Let us say that the computational resources required to parse an unambiguous string of length n is $R(n)$; the total resource commitment to test exhaustively each of the unambiguous strings, and hence to parse the original ambiguous string, is therefore $p^n \times R(n)$. Even for small values of p (in fact even for $p=2$) parsing becomes intractable for all but the shortest of strings. But no linguistic theory, insofar as such things are seriously addressed, has claimed (or would wish to claim) that on the average 'words' are just 2-way ambiguous. A glance at any desktop dictionary should convince anyone that real human languages support considerable ambiguity (also mentioned in Chapter One). Notice also that the order of complexity of this problem is *orthogonal* to the number of ways in which an object can be ambiguous; the problem is $O(np^n)$, that is it increases dramatically with the *length of the input string*. So whether our theory entails twice or ten or even a hundred times as much ambiguity as other theories, it does *not* make the computational complexity of the parsing problem for our theory any different from what it would be for other theories.

Clearly, human beings do parse utterances of varying lengths efficiently in real time, so we must assume that there is some algorithm which is able to disambiguate and parse a string without doing an exhaustive search of all the possibilities.[2] Here is a simple proposal, which we might dub 'divide and conquer': the human linguistic system is a physically finite biological mechanism; as such, certain aspects of its architecture which imply infinitudes or unboundedness are physically constrained by the material tolerances of the device (see the discussion in Chapter One); assume therefore that there is a physical constant k belonging to the human linguistic parsing mechanism such that it can only 'deal with' ambiguous strings no longer than length k; then, given an ambiguous string of length n the device *divides* the string into (n/k) 'chunks' and disambiguates and parses each chunk in turn; the resources required for each chunk are $p^k \times R(k)$, and there are (n/k)

chunks, so the total complexity of this algorithm is $O((n/k)p^kR(k))$; given that k and p are constants of the system, this is equivalent to $O(n)$, *ie.* the algorithm is linear, and therefore tractable. Again, note that the amount of ambiguity (p) in the system does *not* affect the complexity of the parsing algorithm.[3]

4.4 *Acquisition*

Since all information is stored with LObjects in the lexicon, the problem of acquiring syntax reduces to the task of identifying language-particular decompositions, and generalising them. The latter part of this task is precisely the problem of populating the lexicon, which we solved in Chapter Three. In Chapter Three we developed a theory of the acquisition of arbitrary features, and this applies equally to syntactic features as to morphological ones. The theory allows us to populate the lexicon with a predominant virtual structure, while allowing the possibility of the acquisition of idiosyncratic forms, within the definitional parameters of mass-adjustment.

Identifying decompositions follows directly from our choice of definition of $\oplus$ in **S**. Given any assemblage, it should be possible to list every possible decomposition. Let us symbolise the set of all decompositions of a with a^+. In general for an assemblage a with n occurrences of atoms there are $(n-1)$ occurrences of $\oplus$ and $|a^+|=2^{(n-2)}+1$.

Imagine further that a young human being can, with enough prompting, recreate a conceptual-semantic structure corresponding to some piece of the world of its environment, for which there is a syntactic structure k. Assume further that it is able to identify PWords, and that there is some environmental pressure to make an association between a string of PWords $p_1 \ldots p_n$ and k. Acquisitional effort is concentrated in selecting which member of k^+ to store in the lexicon, such that its components are distributed across the $p_1 \ldots p_n$. We can assume that acquisition proceeds recursively, thus minimising the computational complexity of the problem. That is, only the shortest assemblages will be tackled first. Once the basis has been laid, analysis proceeds recursively on that basis.

For example we might learn dɔg *dog*, with syntactic structure d, stored at **d**. Then we might encounter ðəˈdɔg, *the dog*, with syntactic structure $d \oplus t$, to be decomposed and distributed over **d** and **d+t**. The decompositions of $d \oplus t$ are $\{\{d \oplus t\}, \{d\} \oplus \{t\}\}$, and we have two LNodes to fill, so we can deduce that we should store d and t (rather than $d \oplus t$) at **d** and **d+t**. We already have d at **d**, so we can immediately store t under **d+t**.

The storage of the actual choice is accomplished in accordance with the theory of Chapter Three, hence the virtual structure $X \oplus t$ at α_1+**t** will be generated and propagated throughout the lexicon. Thus, future encounters of assemblages involving *t* will already have a strategy for reducing the candidate decompositions, as an analysis of $X \oplus t$ assemblages already exists.

The computational burden of acquisition under our view is therefore $O(2^n)$, for *n* the number of syntactic atoms in a single instance of a learning stimulus. This is clearly only feasible when *n* is very small, hence we need to stipulate that the acquisition device restricts its attention to 'short sentences' (*cf.* §4.3). This in turn implies that all languages will share a great deal of 'topology': LObjects will tend not to contain structure over a certain complexity (we expect not to find many examples in any language of a single LObject stored at ˈblɪk meaning '*Shall I compare thee to a summer's day?*'), and most assemblages will tend to be communicated through a sequence of LObjects containing fairly simply assemblages (probably of no more than two or three atoms). Thus we can use the daunting computational complexity of the acquisition task to our advantage, for we can see in it the need to stipulate a divide and conquer strategy, which in turn provides an interesting explanation of why sentence structure is compositional—it has to be, otherwise we could never acquire language.

The architecture allows for a language where most sentences are stored as single LObjects (as we would expect from *l'arbitraire du signe*). The reason we don't find such languages is *not* because we don't have enough space to store a huge number of sentences (we do, and we still have an efficient means of accessing them, *qv.* Chapter One), but because an organism which takes arbitrarily long assemblages as learning stimuli would die of old age before acquiring the language.

4.5 *The Future*

Looking at it from another perspective we should be able, with a sufficiently explicit syntactic theory, to provide theoretical calculations of the computational resources needed for various values of *n*. Comparing these with a large cross-linguistic study of the level of complexity of lexicalised assemblages should provide us with a good estimate of *n* for natural language. Hence we should have an estimate of the computational power of that part of the brain that is used to acquire language. Ultimately we hope to be able to correlate this quantity directly with measurable quantities of energy and matter in the human brain. Again we see, as we have throughout this study, a simple, general mechanism which is constrained by some physical tolerance, which we can represent as a physical constant of our theory. In this sense our theory is not at all 'unusual', and makes the mechanisms of language acquisition and processing look not unlike the physical systems we are accustomed to describing elsewhere in Nature.

Additionally, our theory has implications beyond the narrow scope of language, for it claims that properties of language processing can provide data on the values of certain physical constants related to the upper bounds of computational power available to the human brain.

4.6 *Notes to Chapter Four*

1. We remind ourselves that there is a small class of contact phenomena associated with the acoustic transmission of speech. As discussed in Chapter Two, these processes have rather different properties from those to be considered in this chapter. They are subconscious, automatic, exceptionless, and crucially, non-transformational. They include so-called nasal place assimilation, a particularly prevalent process in connected speech.
2. This sort of problem has been at the centre of Artificial Intelligence research since its first days, where it arose in connection with the design of chess-playing computer programs (Newell, Shaw & Simon 1995, Minsky 1995).
3. The physical (or biological) constants *p* and *k* are particular properties of the human linguistic system, whose values will have been determined by evolution as those able to exploit in some optimal way the computational resources available in the brain. For similar considerations in relation to constants associated with the human linguistic system compare the discussion of the order of the human linguistic hashing system (Chapter One) and the length restrictions on primary and secondary keys (Chapter Two).

Appendix A

Hashing Systems

The purpose of this appendix is to give a more formal analysis of those aspects of hashing systems discussed in the main text. We consider the general hashing system $\mathfrak{H}(n)$, which consists, by definition, of n layers of keys, one of which is in a one-to-one mapping with records and their addresses. In such a system, each key layer i, $0<i\leq n$ consists of ${}^{n}k_i$ keys, and each layer has ${}^{n}p_i$ 'processing units' per key dedicated to it.

The resources required for each layer i are $R_i = S({}^{n}k_i{}^{n}p_i) + T({}^{n}p_i^{-1})$. That is, space is needed for each of the processing units, and the processing units divide the temporal resources equally. We let the processing units define the units of space and time. Thus one processing unit takes up one unit of space, and each processor requires one unit of time to perform its operation. Therefore, for any layer, the spatial resources must always be an integer multiple of one unit of space, and the temporal resources must be an integer multiple of one unit of time (A.1)

A.1 Lower bounds on resources

For a hashing system $\mathfrak{H}(n)$, for any layer i, $0<i\leq n$, with appropriate choice of spatial and temporal units, the resource commitment R_i for layer i is $R_i = {}^{n}k_i{}^{n}p_i + {}^{n}p_i^{-1}$, where ${}^{n}k_i{}^{n}p_i \geq 1$ and ${}^{n}p_i^{-1} \geq 1$, (for $n, {}^{n}k_i, {}^{n}k_i{}^{n}p_i$ and ${}^{n}p_i^{-1}$ integers).

The total resource commitment, $R(n)$, for the entire system $\mathfrak{H}(n)$ is simply the sum of all the resource commitments for each layer, namely, $R(n) = R_1 + \ldots + R_n$. This expands straightforwardly to $R(n) = S({}^{n}k_1{}^{n}p_1 + \ldots + {}^{n}k_n{}^{n}p_n) + T({}^{n}p_1^{-1} + \ldots + {}^{n}p_n^{-1})$. Let us introduce the following notation to ease the exposition: let $\Sigma^{n}\alpha_i =_{df} {}^{n}\alpha_1 + \ldots + {}^{n}\alpha_n$. The total resource commitment is therefore (A.2)

A.2 Total resource commitment

For a hashing system $\mathfrak{H}(n)$ with layers i, $0<i\leq n$, the total resource commitment $R(n)$ is given by $R(n) = \Sigma^{n}R_i$. With appropriate choice of spatial and temporal units we have equivalently $R(n) = \Sigma^{n}k_i{}^{n}p_i + \Sigma^{n}p_i^{-1}$.

Because the system models a 'biological mechanism', there is a fixed amount of resources available. This tolerance is given by two constants A and B, where A is the spatial tolerance, and B the temporal tolerance. The total resource commitment can never exceed these limits, thus $R(n) \leq S(A) + S(B)$. By equating the spatial and temporal components of this relation with the total resource commitment given in A.2 we derive A.3.

A.3 Upper bounds on resources

For a hashing system $\mathfrak{H}(n)$ with layers i, $0<i\leq n$, for spatial tolerance A and temporal tolerance B, with appropriate choice of spatial and temporal units, $A=\Sigma^n k_i{}^n p_i$ and $B=\Sigma^n p_i^{-1}$.

From A.1 we have ${}^n p_i^{-1}\geq 1$, hence ${}^n p_i\leq 1$. Given A.3 we have, therefore, $A\geq\Sigma^n k_i$ and $B\geq n$ (since B is equal to the sum of n numbers, each of which is greater than or equal to 1). Thus we have that the order of the hashing system is constrained by the temporal resources of the mechanism, and the number of keys is constrained by the spatial resources (A.4).

A.4 Constraints on keys and orders

For a hashing system $\mathfrak{H}(n)$ with layers i, $0<i\leq n$, for spatial tolerance A and temporal tolerance B, with appropriate choice of spatial and temporal units, $A\geq\Sigma^n k_i$ and $B\geq n$.

The total number of records accessible by the system, $D(n)$, is the product of the number of keys in each layer, namely $D(n)={}^n k_1\times\ldots\times{}^n k_n$. Again let us define a convenient notation: let $\Pi^n\alpha_i =_{df} {}^n\alpha_1\times\ldots\times{}^n\alpha_n$. We have A.5.

A.5 Total addressable records

For a hashing system $\mathfrak{H}(n)$ with layers i, $0<i\leq n$, the total number of accessible records $D(n)$ is given by $D(n)=\Pi^n k_i$.

This number, $D(n)$, of accessible records is obviously maximised when each of the factors ${}^n k_i$ of $D(n)$ is maximised. However, from A.4 we know that the number of keys is constrained such that $A\geq\Sigma^n k_i$. Thus the maximum values of the ${}^n k_i$ are the maximum values satisfying $A=\Sigma^n k_i$. It can be shown that these values are maximised if and only if they divide A equally; that is, if there is some integer K such that ${}^n k_i=K$. Given $A=\Sigma^n k_i$ we have $A=nK$, hence ${}^n k_i=A/n$. From A.5 it follows that the maximum addressable number of records is $(A/n)^n$. (A.6).

A.6 MAXIMUM ADDRESSABLE RECORDS

For a hashing system $\mathfrak{H}(n)$ with spatial tolerance A, and appropriate choice of spatial units, the maximum number of accessible records $max(D(n))$ is given by $max(D(n))=(A/n)^n$.

If we choose an order one system, $\mathfrak{H}(1)$—a system where the keys are mapped one-to-one to addresses, as illustrated in (3)—we have $max(D(1))=A$. We can therefore think of the spatial tolerance A as the maximum size of a database accessible by the simple non-hashing device illustrated in (3). Given that n is constrained by the spatial tolerance B (A.4 gives us $B \geq n$), what values of n maximise the maximum possible number of accessible records? That is, what value of n gives $max(max(D(n))$?

Analysis of the function $max(D(n))=(A/n)^n$ by the usual methods of differential calculus reveals that its maximum occurs when $A=ne$, giving a maximum possible database size of $e^{(A/e)}$, using an $\mathfrak{H}(A/e)$ system.[1] Given that both A and n must be integers, and e is irrational, this maximum is never achievable in practice. Let n_1 be the largest integer no greater than A/e; let n_2 be the smallest integer greater than A/e (in fact $n_2=n_1+1$); the maximum achievable database size is given by $n=n_1$ if and only if $max(D(n_1))>max(D(n_1+1))$, otherwise $n=n_1+1$. Now, $max(D(n_1))>max(D(n_1+1))$ implies $(A/n_1)^{n_1}>(A/(n_1+1))^{(n_1+1)}$. But this inequality is trivially true, for any values of A and n_1. Therefore the maximum achievable database size will always be when $n=n_1$ (A.7).

A.7 OPTIMAL VALUE OF n

For a hashing system $\mathfrak{H}(n)$ with spatial tolerance A, and temporal tolerance B, with appropriate choice of spatial and temporal units, the value of n which gives the largest achievable maximum number of accessible records is the largest integer satisfying both $n \leq B$ and $n < A/e$.

Finally we note that given tolerances A and B, there are systems of order $\mathfrak{H}(n)$, with databases of size $D(n)$ which cannot be accessed by any $\mathfrak{H}(n-1)$ system, where $max(D(n-1))<D(n)$. From A.5 and A.6 we have, therefore that if the product of the number of keys in each layer of the $\mathfrak{H}(n)$ system lies between $max(D(n-1))$ and $max(D(n))$ then the system can access databases larger than any database accessible by an $\mathfrak{H}(n-1)$ system, with the same tolerances. Expanding the *max* expressions gives A.8.

A.8 Database sizes increasing with hashing order

For given tolerances A and B, with appropriate choice of units, for a system $\mathfrak{H}(n)$, with layers i, $0<i\leq n$, and ${}^{n}k_i$ keys in layer i, if $(A/(n-1))^{(n-1)}<\Pi^{n}k_i\leq(A/n)^{n}$, then the system can access databases larger than any database accessible by an $\mathfrak{H}(n-1)$ system, working within the same tolerances A and B.

Notes to Appendix A

1. where e is the base of natural logarithms, $e\approx 2.718$. The derivative, or gradient function, of $(A/n)^n$ is $\ln(A/n)-1$. That is, $(max(D(n)))'=\ln(A/n)-1$. The turning point therefore occurs at $\ln(A/n)-1=0$, namely when $\ln(A/n)=1$, equivalently, $A=ne$. The gradient $(max(D(n)))'$ is positive up to this point, and negative after it, so the turning point is a maximum.

Appendix B

Hashkey Implementation

The basic insight behind the hash key implementation is that there should be a one-to-one mapping from phonological keys to natural numbers, each natural number in the range of the function being a primary hash key in the Natural Language hash table (Chapter One). Now, this mapping could in principle be arbitrary—each phonological key is assigned arbitrarily one of the numbers in the hash key range. However, a set of integers (of which the hash keys is one) has *structure*. We can say, for example, that integer *a* precedes integer *b*, or that integer *c* is the sum of *a* and *b*, etc.

With this is mind, we define the mapping from phonological keys to integers in a non-arbitrary way. Specifically we adhere to the principle that semantic models are built *compositionally* from the structures they interpret/implement (Tarski 1965). Accordingly, the phonological categories of a phonological key are considered the 'digits' of the number which is its implementation. Each phonological category can contain any one of seven categories $\kappa = \{\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D},\mathbf{E},\mathbf{F},\mathbf{G}\}$ and so there are a possible $2^7 = 128$ 'digits' in any one of the four positions of the phonological key. We can therefore set up a simple mapping from phonological keys to four-digit integers in base 128.

B.1 Hash Key Implementation (1)
There is an interpretation function [[ ]] such that
$[[\ ]] : \mathbf{K}^4 \rightarrow \{\textit{Integers } n \mid 0 \leq n \leq 128^4 - 1\ (=268{,}435{,}455)\}$

This in turn requires that we set up a mapping from all possible phonological categories to the 'digits' 0–127. Again, this is simply achieved by mapping the subsets into 7-digit binary numbers, where each binary digit (bit) corresponds to a category—1 if the category is a member of the phonological category, 0 if it isn't.

B.2 Hash Key Implementation (2)[1]
There is a function $I : \wp(\kappa) \rightarrow \{\textit{Integers } n \mid 0 \leq n \leq 127\}$ such that for any phonological category x in $\wp(\kappa)$,

$I(x) =_{df} (g\times 2^6)+(f\times 2^5)+(e\times 2^4)+(d\times 2^3)+(c\times 2^2)+(b\times 2^1)+(a\times 2^0)$
where $a=1$ iff $\mathbf{A}\in x$; $a=0$ iff $\mathbf{A}\notin x$
$b=1$ iff $\mathbf{B}\in x$; $b=0$ iff $\mathbf{B}\notin x$
$c=1$ iff $\mathbf{C}\in x$; $c=0$ iff $\mathbf{C}\notin x$
$d=1$ iff $\mathbf{D}\in x$; $d=0$ iff $\mathbf{D}\notin x$
$e=1$ iff $\mathbf{E}\in x$; $e=0$ iff $\mathbf{E}\notin x$
$f=1$ iff $\mathbf{F}\in x$; $f=0$ iff $\mathbf{F}\notin x$
$g=1$ iff $\mathbf{G}\in x$; $g=0$ iff $\mathbf{G}\notin x$

We can now state the definition of [[ ]] completely.

B.3 Hash Key Implementation (3)
There is an interpretation function [[ ]] such that
$[[\ ]] : \mathbf{K}^4 \rightarrow \{\textit{Integers } n \mid 0\leq n\leq 128^4-1\}$ where for some phonological key $\delta=(x_0,x_1,x_2,x_3)$ in $\mathbf{K}^4$,
$[[d]] =_{df} (I(x_0)\times 128^3)+(I(x_1)\times 128^2)+(I(x_2)\times 128^1)+(I(x_3)\times 128^0)$

Recall finally that we wish affixes to be interpreted as *offsets*, that is, integers. The most straightforward way of doing this is to use the same strategy as was used for generating an integer for each phonological key in the definition of [[ ]] above. We therefore reuse the definition almost *verbatim*.

B.4 Hash Key Implementation of Affixes
For some affix ξ in $\mathbf{K}^2$, where $\xi=(x_0,x_1)$,
$[[\xi]] =_{df} (I(x_0)\times 128^1)+(I(x_1)\times 128^0)$

We can now state the definition of the (hash key) interpretation of a PWord:

B.5 Hash Key Implementation of PWords
The hash key interpretation of a PWord $\pi\in(\mathbf{K}^2)^n\times\mathbf{K}^4\times(\mathbf{K}^2)^m$,
written $[[\pi]]$, where $\pi\times(\xi_1,\ldots,\xi_n,\delta,\zeta_1,\ldots,\zeta_m)$, and $\delta=[[\delta]]$ is
$[[\pi]]=(\delta+[[\zeta_m]],\ldots,\ \delta+[[\zeta_1]],\delta,\delta+[[\xi_n]],\ldots,\delta+[[\xi_1]])$.

Note that the total hash table space defined by this implementation is the range of integers $0\leq n<128^6$. This is equivalent to the range $0\leq n<(2^7)^6$, or, $0\leq n<2^{42}$. The addressing system for Natural Language is therefore a 42-bit one. Most of today's desktop PCs use 32-bit addressing, and the growing range of Pentium chips already uses 64-bit internal addressing. Workstation computers have been using 64-bit addressing for some time, so an efficient 'silicon' implementation of the Natural Language hash key system using 'off-the-shelf' technology is certainly feasible.

Let us pursue a simple example in detail. In the example we represent integers in base 128, thus 0.33.20 is used to represent $(0\times128^2)+(33\times128)+20=4{,}244$.

Consider the PWord (34). The interpretation of this PWord is a sequence of hash keys, all at various offsets (given by the interpretations of the bound forms) from the primary hash key (the interpretation of the phonological key) (35).

(34) *A PWord*

((**GFB**,**EC**),(**0**,**FEDC**,**GFDCA**,**FDA**),(**GFEC**,**GFCA**))

(35) *Hash Key Implementation of* (34)

The implementation of the prefix is [[(**GFB**,**EC**)]]=98.20

The implementation of the phonological key is
[[(**0**,**FEDC**,**GFDCA**,**FDA**)]]=0.60.109.41
The bound forms are therefore offsets from
$0.60.109.41\times128^2=0.60.109.41.0.0$

The implementation of the suffix is [[(**GFEC**,**GFCA**)]]=116.101

The implementation of the PWord in (34) is the sequence
(0.60.109.41.0.0+116.101,0.60.109.41.0.0,0.60.109.41.0.0+98.20)
=(0.60.109.41.116.101,0.60.109.41.0.0,0.60.109.41.98.20),
representing three lexical look-ups.

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